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Effect of microwave radiation on the nonlinear resistivity of a two-dimensional electron gas at large filling factors

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We study transport properties of a two-dimensional electron gas, placed in a classically strong perpendicular magnetic field and in constant and oscillating in-plane electric fields. The analysis is based on a quantum Boltzmann equation derived for a weakly disordered two-dimensional electron gas. We consider disordered potential with both long- and short-range correlations. Electron scattering off such disorder is not limited to small change in momentum direction, but occurs on an arbitrary angle, including the backscattering. The nonlinearity of the transport in the considered system is a consequence of two coexisting effects: formation of a nonequilibrium distribution function of electrons and modification of the scattering rate off the disorder in the presence of dc and ac electric fields. This work describes both effects in a unified way. The calculated dissipative component of electric current oscillates as a function of the electric-field strength and frequency of microwave radiation in qualitative agreement with experiments.

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I. INTRODUCTION

The discovery of the microwave-induced resistance oscillations (MIROs) (Refs. 1 and 2) and the zero-resistance states (ZRSs) (Refs. 3–6) has raised interest in nonlinear transport properties of two-dimensional electron systems (2DESs) in a perpendicular magnetic field at large filling factors. The dissipative dc magnetoresistance exhibits giant oscillations with the inverse magnetic field when exposed to a microwave radiation.[1,2,7–12] The period of MIRO is controlled by the ratio of the microwave frequency ω to the electron cyclotron frequency $\omega_c = |e|B/mc$ in magnetic field B. In the high-mobility samples the MIROs evolve into the ZRS when the linear dc resistance becomes negative.[13,14] These remarkable findings made it imperative to understand the mechanism of oscillations preceding the onset of the ZRS. The experiments were performed at relatively high temperatures and low magnetic fields, when the Shubnikov–de Haas oscillations are suppressed.

The appearance of the MIRO has been first attributed to the modification of the impurity scattering rates in the presence of the magnetic field.[15–17] This scenario is commonly referred to as the "displacement" mechanism. It has been predicted[18,19] that a different so-called inelastic mechanism dominates in the regime where both MIRO and ZRS were observed.[3–6] According to Ref. 18, the microwave radiation is responsible for formation of a nonequilibrium component of the distribution function, which is isotropic in momentum and oscillates as a function of energy. The amplitude of such nonequilibrium component of the distribution function is characterized by the temperature-dependent rate of inelastic-scattering processes $1/\tau_{ee}$ due to the electron-electron interaction. The analysis of Ref. 18 suggests that in weak electric fields and at sufficiently low temperatures, the "inelastic" contribution from the nonequilibrium component of the distribution function to the linear-response dc resistivity is larger than the "displacement" contribution.

A different series of experiments focused on measurements of the nonlinear differential resistance in the absence of microwave excitation.[20–25] In these experiments, the differential resistance has been measured in the Hall bar geometry as a function of the applied direct current. This current creates a strong electric field in a perpendicular direction, known as the Hall field, provided that the magnetic field is strong, $\omega_c \tau_{tr} \gg 1$, where τ_{tr} is the transport scattering time. The scattering off disorder in the Hall field is accompanied by a change in electron kinetic energy and leads to dependence of transport characteristics on the strength of this field. In particular, the differential resistance exhibits oscillations, called the Hall induced resistance oscillations (HIROs), as a function of the Hall electric-field strength E. The HIROs were explained[20] as a result of the geometric resonance in the electron transitions between the tilted Landau levels when the diameter of the cyclotron trajectory becomes commensurable with the spatial modulation of the density of states. More rigorous approach[26] employing the quantum kinetic equation showed that the inelastic mechanism is important in a relatively narrow interval of applied electric fields, and the displacement mechanism becomes dominant in the regime of strong direct current where HIRO were observed.

The effect of the microwave irradiation on the nonlinear transport was experimentally investigated in Refs. 27 and 28, in which a 2DES was subject to both constant and oscillating electric fields. The value of the differential magnetoresistance depends on two dimensionless parameters,

$$\epsilon_{dc} = \frac{|e|E(2R_c)}{\omega_c}, \quad \epsilon_{ac} = \frac{\omega}{\omega_c}, \tag{1.1}$$

where E is the magnitude of the in-plane constant electric field, $R_c = v_F/\omega_c$ is the cyclotron radius, v_F is the Fermi velocity, and ω_c is the cyclotron frequency; throughout this paper we use $\hbar = 1$. Maxima of the magnetoresistance in the vicinity of the main diagonal of the two-dimensional

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$(\epsilon_{ac}, \epsilon_{dc})$ plane are obtained when the sum $\epsilon_{ac}+\epsilon_{dc}$ is integer. Interestingly, this simple rule does not hold farther away from the main diagonal $\epsilon_{dc}\sim\epsilon_{ac}$. In fact, the interplay between both types of excitation gives rise to an unexpectedly rich structure of extremes and saddle points of the differential magnetoresistance in the $(\epsilon_{ac}, \epsilon_{dc})$ plane.[27]

Oscillations of the differential resistance as a function of ϵ_{dc} are understood[20–22,26] in terms of electron backscattering off impurities, which corresponds to change in electron direction on its opposite. Therefore, an appropriate model of disordered potential has to include processes of electron scattering on an arbitrary angle θ, including $\theta=\pi$. Such potential has both long-range correlations being responsible for small-angle scattering, and short-range correlations leading to backscattering. The proper treatment of the disorder potential with above properties requires a further extension of a kinetic theory of 2DES (Ref. 17) developed for smooth disorder.

The goal of the present paper is to construct a systematic theory of magneto-oscillations of the differential resistivity of the 2DES in the presence of electric fields of arbitrary strength. In experiments of Refs. 27 and 28, oscillations have been observed at large filling factors, $\sim E_F/\omega_c \gg 1$, with E_F being the Fermi energy. In this limit, we can treat the kinetics of electron gas semiclassically. The analysis is performed in the experimentally relevant range of classically strong magnetic fields, $1/\tau_{tr} \ll \omega_c$. In the present work we focus on the situation when Landau levels are not resolved, which implies the inequality $\omega_c\tau_q \lesssim 1$, where τ_q is the quantum-scattering time. The temperature is assumed to be relatively large, $T \gtrsim \omega_c$, so that the Shubnikov–de Haas oscillations are exponentially suppressed ($k_B=1$).

The two most important contributions to the nonlinear electric current are the inelastic contribution originating from the modification of the electron distribution function[8,18,26,29] and the displacement contribution representing the changes of electron-scattering amplitudes off disorder. Additional contributions were identified and studied in Ref. 29, but these contributions have additional smallness in systems with mixed disorder. The displacement mechanism can be studied by various methods,[15–17,30–33] which provide a qualitatively correct picture for electron transport in strong electric fields. However, for a full description of the crossover from "weak" to "strong" fields, the kinetic equation is necessary.

The paper is organized as follows. In Sec. II we present a simplified analysis of magneto-oscillations in combined constant and oscillating electric fields and summarize the main results. In Sec. III the kinetic equation is derived in the framework of the Keldysh formalism. We solve the kinetic equation within a bilinear response in microwave field and apply this solution to calculation of the nonlinear current in Sec. IV. Section V contains an analysis of the current beyond the bilinear in microwave field response. Discussion and conclusions are presented in Sec. VI.

II. MAIN RESULTS

A. Bilinear response to the applied microwave radiation in strong dc electric field

In this section we present heuristic discussion of the results of the paper for the dissipative current in strong dc electric field, but consider the contribution to the electric current that is only bilinear in the microwave electric field. Our analysis employs the semiclassical treatment of electron motion in crossed electric and magnetic fields, valid in the limit of large filling factor $E_F/\omega_c \gg 1$, where E_F is the Fermi energy and ω_c is a cyclotron frequency. According to this picture electron scattering off impurities amounts to the spatial shift of the guiding center of the cyclotron orbit (Fig. 1).

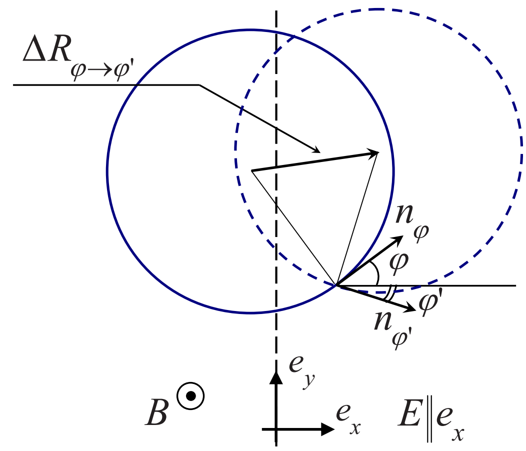

FIG. 1. (Color online) Scattering off impurity leads to the shift of the guiding center of a cyclotron electron trajectory.

In the presence of the electric, $\boldsymbol{E}=\boldsymbol{e}_x E$, and magnetic, $\boldsymbol{B}=\boldsymbol{e}_z B$, fields, the dissipative current results from the imbalance between the drift of cyclotron orbits parallel to the electric field. We denote a unit vector forming angle φ with the direction of the electric field $\boldsymbol{e}_x$ by $\boldsymbol{n}_\varphi=\{\cos\varphi;\sin\varphi;0\}$. The electron scattering resulting in the change in the direction of motion from $\boldsymbol{n}_\varphi$ to $\boldsymbol{n}_{\varphi'}$ leads to the shift of the guiding center $\Delta\boldsymbol{R}$ given by

$$\Delta\boldsymbol{R}_{\varphi\to\varphi'} = R_c \boldsymbol{e}_z \times (\boldsymbol{n}_\varphi - \boldsymbol{n}_{\varphi'}). \tag{2.1}$$

We present the current as a sum of two contributions

$$j = j_1 + j_2. \tag{2.2}$$

Here, the first term

$$j_1 = 2e\int \frac{d\varphi d\varphi'}{(2\pi)^2}\int dx \int d\varepsilon\, \nu(\varepsilon,x)\Gamma^1_{\varphi\to\varphi'} \times[f(\varepsilon,x) - f(\varepsilon,x+\boldsymbol{e}_x\Delta\boldsymbol{R}_{\varphi\to\varphi'})] \tag{2.3}$$

describes the current in the absence of processes changing electron energy by absorption or emission of a microwave field quantum with energy ω. This term contains virtual processes of electron scattering in microwave field, which modify momentum scattering rate off disorder. Thus, function $\Gamma^1_{\varphi\to\varphi'}$ is the disorder scattering rate for the direction of electron momentum from $\boldsymbol{n}_\varphi$ to $\boldsymbol{n}_{\varphi'}$ and has the following form within the Born approximation:

$$\Gamma^1_{\varphi\to\varphi'} = \left[\frac{1}{\tau_{\varphi-\varphi'}} - \frac{\mathcal{P}_\omega}{\bar{\tau}_{\varphi-\varphi'}}\right]\frac{\nu(\varepsilon, x+\Delta X_{\phi\to\phi'})}{\nu_0}, \qquad (2.4)$$

where $1/\tau_{\varphi-\varphi'}$ is the disorder scattering rate in the absence of electric and magnetic fields, and

$$\frac{1}{\bar{\tau}_{\varphi-\varphi'}} = \frac{1-\cos(\varphi'-\varphi)}{\tau_{\varphi-\varphi'}}. \qquad (2.5)$$

The dimensionless parameter

$$\mathcal{P}_\omega = \frac{v_F^2 e^2 E_\omega^2}{2\omega^2(\omega\mp\omega_c)^2} \qquad (2.6)$$

is proportional to the microwave power. In this paper we limit the consideration to the circularly polarized microwave radiation. The upper (lower) sign in Eq. (2.6) corresponds to the right (left) circular polarization of the incoming microwave radiation. The anisotropy of transport coefficients in the case of arbitrary microwave polarization is left for a separate investigation.[34] Close to the cyclotron resonance the dynamical screening of the microwave field by electron system has to be taken into account. This screening results in a modified form of parameter $\mathcal{P}_\omega$ (see Ref. 35). The drift of the guiding centers due to the microwave field effectively smears the disorder potential felt by the electron. For that reason the scattering rate $\Gamma^1_{\varphi\to\varphi'}$, Eq. (2.4), is suppressed by the microwave radiation. This scattering rate suppression is reminiscent of the "motion narrowing" phenomenon and is further discussed in Sec. III C.

Apart from the virtual processes of electron interaction with microwave field, there are real processes, which are accompanied by the absorption and emission of photons. The second term in Eq. (2.2) takes into account the contribution to the electric current from such processes,

$$j_2 = 2e\sum_{\pm}\int\frac{d\varphi d\varphi'}{(2\pi)^2}\int dx\int d\varepsilon\,\nu(\varepsilon,x)\Gamma^{\pm}_{\varphi\to\varphi'} \times[f(\varepsilon,x) - f(\varepsilon\pm\omega, x+\boldsymbol{e}_x\Delta\boldsymbol{R}_{\varphi\to\varphi'})]. \qquad (2.7)$$

The rate of such scattering processes can be written in the form

$$\Gamma^{\pm}_{\varphi\to\varphi'} = \mathcal{P}_\omega\frac{\nu(\varepsilon\pm\omega, x+\Delta X_{\varphi\to\varphi'})}{\nu_0\bar{\tau}_{\varphi-\varphi'}}. \qquad (2.8)$$

We emphasize that in the present subsection we do not consider multiphoton processes. This approximation is justified if the dimensionless parameter $\mathcal{P}_\omega \ll 1$. We analyze the case of the arbitrary parameter $\mathcal{P}_\omega$ in Sec. V (see also Sec. II B).

The angular integrals in Eqs. (2.3) and (2.7) are restricted by the conditions $\Delta X_{\varphi\to\varphi'} = \boldsymbol{e}_x\Delta\boldsymbol{R}_{\varphi\to\varphi'} > 0$, and spatial integration is limited to the stripe $-\Delta X_{\varphi\to\varphi'} < x < 0$. The electron density of states has spatial dependence in constant electric fields. This dependence appears under the spatial integral and we discuss this dependence in more detail. In a perpendicular magnetic field and in the absence of electric fields, the density of states of a 2DES, $\nu(\varepsilon)$, has an energy modulation with period ω_c,

$$\nu(\varepsilon) = \nu_0\left(1 - 2\lambda\cos\frac{2\pi\varepsilon}{\omega_c}\right), \qquad (2.9a)$$

where ν_0 is the density of states in the absence of fields and the factor $\lambda = \exp(-\pi/\omega_c\tau_q) \lesssim 1$. A constant electric field tilts electron density of states along its direction, $\boldsymbol{e}_x$, resulting in spatial dependence of the density of states,

$$\nu(\varepsilon,x) = \nu(\varepsilon + eEx). \qquad (2.9b)$$

We present the results for the nonlinear current in strong dc electric fields, $\epsilon_{dc} = 2|e|ER_c/\omega_c \gg 1$, where the differential resistance exhibits an oscillatory behavior. Rigorous analysis of Sec. IV C shows that in this case the dominant contribution to the nonlinear current originates from the smooth component of the distribution function, which can be taken as the Fermi-Dirac distribution function at temperature T,

$$f_T(\varepsilon) = \frac{1}{e^{\varepsilon/T}+1}. \qquad (2.10)$$

Substitution of Eq. (2.4) into Eq. (2.3) gives the contribution j_1 to the current in the form

$$j_1 = \frac{2e}{\nu_0}\int\frac{d\varphi d\varphi'}{(2\pi)^2}\int d\varepsilon\,\nu(\varepsilon)\nu(\varepsilon+eE\Delta X_{\varphi\to\varphi'})\Delta X_{\varphi\to\varphi'} \times\left[\frac{1}{\tau_{\varphi-\varphi'}} - \frac{\mathcal{P}_\omega}{\bar{\tau}_{\varphi-\varphi'}}\right][f_T(\varepsilon) - f_T(\varepsilon+eE\Delta X_{\varphi\to\varphi'})]. \qquad (2.11)$$

From Eq. (2.11) we obtain

$$j_1 = \sigma_D E + \delta j_1^{(2)}. \qquad (2.12)$$

Here the first term is the zeroth order in λ, neglecting the oscillations of electron density of states in magnetic fields in Eq. (2.11). This term corresponds to the classical Drude contribution to the current at large Hall angles,

$$\sigma_D = e^2\nu_0\frac{R_c^2}{\tau_{tr}}, \quad \frac{1}{\tau_{tr}} = \int_{-\pi}^{+\pi}\frac{d\theta}{2\pi}\frac{1-\cos\theta}{\tau_\theta}, \qquad (2.13)$$

where τ_{tr} is the transport scattering time. The second term in Eq. (2.12) represents a contribution to the current, which is nonlinear in the applied electric field E and quadratic in parameter λ,

$$\delta j_1^{(2)} = 2\lambda^2 e^2\nu_0 E\int\frac{d\varphi d\varphi'}{(2\pi)^2}\left[\frac{1}{\tau_{\varphi-\varphi'}} - \frac{\mathcal{P}_\omega}{\bar{\tau}_{\varphi-\varphi'}}\right] \times[\Delta X_{\varphi\to\varphi'}]^2\cos\frac{2\pi eE\Delta X_{\varphi\to\varphi'}}{\omega_c}. \qquad (2.14)$$

The contribution of the first order in λ, which is omitted in Eq. (2.12), describes the Shubnikov–de Haas oscillations and can be estimated as $\propto\sigma_D\lambda_T\lambda$. Here the additional small prefactor $\lambda_T = \exp(-2\pi^2 T/\omega_c)$ appears as a result of averaging of the rapid oscillations in the density of states over thermal energies $|\varepsilon| \lesssim T$. The contribution of the second order in λ contains a square of the oscillating component of the density of states and is not exponentially suppressed after integration over thermal energy window. In this paper we consider the limit of relatively high temperatures $T \gg \omega_c/2\pi$. The latter condition is normally satisfied in experiments performed at the nonlinear regime in dc electric field. Under this condition the quadratic in λ contribution dominates over the Shubnikov–de Haas contribution linear in λ.

For $\epsilon_{dc} \gg 1$ the angular integrations in Eq. (2.14) can be performed in the stationary phase approximation. The main contribution comes from the scattering within narrow intervals centered at $\varphi = \pm\pi/2$ and $\varphi' = \mp\pi/2$. These backscattering processes correspond to the $2R_c$ jumps of electron guiding center along the electric field. Equation (2.14) yields[26]

$$\delta j_1^{(2)} \approx (2\lambda)^2(1-2\mathcal{P}_\omega)\frac{e\nu_0 v_F}{\pi^2\tau_\pi}\sin 2\pi\epsilon_{dc}. \quad (2.15)$$

A similar analysis of the contribution Eq. (2.7) to the electric current due to the real processes of absorption and emission of microwave photons gives

$$j_2 \approx (2\lambda)^2\mathcal{P}_\omega\frac{e\nu_0 v_F}{\pi^2\tau_\pi}\left[\frac{\epsilon_{dc}+\epsilon_{ac}}{\epsilon_{dc}}\sin 2\pi(\epsilon_{dc}+\epsilon_{ac}) + \frac{\epsilon_{dc}-\epsilon_{ac}}{\epsilon_{dc}}\sin 2\pi(\epsilon_{dc}-\epsilon_{ac})\right]. \quad (2.16)$$

We emphasize that both Eqs. (2.15) and (2.16) are expressed through the parameter $\epsilon_{dc} \propto ER_c$. These two expressions are obtained within a stationary phase approximation and correspond to processes representing shifts of electron cyclotron trajectories by distance $2R_c$ along the applied dc electric field. Therefore, Eqs. (2.15) and (2.16) have a simple geometrical interpretation in terms of commensurability between the space modulation of electron density of states by electric field and the maximal displacement $2R_c$ in a single-scattering process off disorder. The expression for the nonlinear current beyond the saddle-point approximation is evaluated in Sec. IV B.

In general, we represent the total current as a sum of the linear Drude term $\sigma_D E$ and the nonlinear term δj, which arises due to oscillatory density of states in perpendicular magnetic field,

$$j = \sigma_D E + \delta j. \quad (2.17)$$

For the overlapping Landau levels, when the density of states is given by Eq. (2.9a), the nonlinear contribution is quadratic in parameter λ.

Within a bilinear response to the applied microwave electric field and in strong dc electric fields, we combine Eqs. (2.2) and (2.12) and obtain

$$\delta j = \delta j_1^{(2)} + j_2. \quad (2.18)$$

Equations (2.15), (2.16), and (2.18) represent some of the main results of this paper. As we discuss in Sec. IV, despite its simplicity, Eq. (2.16) explains pronounced features of the transport measurement reported in Refs. 27 and 28.

B. Current at arbitrary microwave powers in strong dc electric field

The result of Eq. (2.18) is just a limiting case of expression obtained in Sec. V for the current in strong dc electric fields. In this case we again can neglect the modification of the electron distribution function by electric fields and consider only the displacement mechanism for generation of the nonlinear component δj of the current Eq. (2.17). We found the following expression:

$$\delta j = (2\lambda)^2\frac{|e|\nu_0 v_F}{\pi^2\tau_\pi}\left[\sin 2\pi\epsilon_{dc}J_0(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac}) + \frac{2\epsilon_{ac}}{\epsilon_{dc}}\cos 2\pi\epsilon_{dc}\cos\pi\epsilon_{ac}\sqrt{\mathcal{P}_\omega}J_1(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac})\right]. \quad (2.19)$$

Here $J_n(x)$ are the Bessel functions. Performing an expansion in Eq. (2.19) to the first order in $\mathcal{P}_\omega$, we recover Eq. (2.18) in terms of Eqs. (2.15) and (2.16).

C. Weak electric fields

Above results were obtained under assumption that the distribution function of electrons is given by the Fermi distribution function [see Eq. (2.10)]. In the presence of electric fields the distribution function deviates from the equilibrium configuration. This nonequilibrium distribution function affects significantly the electric current in sufficiently weak electric fields. For smooth disorder, we recover the result of Ref. 18 for the nonlinear contribution δj to current in terms of dimensionless parameters ϵ_{dc}, ϵ_{ac}, defined by Eq. (1.1), and $\mathcal{P}_\omega$, defined by Eq. (2.6),

$$\delta j = 2\lambda^2\sigma_D E\frac{\pi^2\epsilon_{dc}^2 + 2\pi\epsilon_{ac}\mathcal{P}_\omega\sin 2\pi\epsilon_{ac}}{\tau_{tr}/\tau_{ee} + \pi^2\epsilon_{dc}^2/2 + 2\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}}. \quad (2.20)$$

Here $1/\tau_{ee}$ is the relaxation rate of the nonequilibrium component of the electron distribution function due to electron-electron interaction. In the presence of sharp disorder, the displacement contribution, arising due to the modification of electron-scattering rate off disorder, may become comparable to the inelastic contribution Eq. (2.20). The latter contribution survives only in relatively weak electric fields, $\epsilon_{dc} \lesssim 1$. In stronger electric fields, $\epsilon_{dc} \gg 1$, it contains an extra small factor $\tau_q/(\tau_\pi\epsilon_{dc})$ and the displacement mechanism becomes more important (see Sec. IV C).

III. QUANTUM KINETIC EQUATION

In the present section, we study a disordered two-dimensional electron gas (2DEG) subject to in-plane electric fields and derive the quantum kinetic equation, following Ref. 17, but consider a mixed disorder characterized by scattering amplitude which is finite for an arbitrary scattering angle. The Dyson equation for the disorder averaged electron Green's function is

$$(i\partial_t - \hat{H})\hat{G}(tt') = \hat{1}\,\delta(t-t') + \int dt_1 \hat{\Sigma}(tt_1)\hat{G}(t_1t'), \quad (3.1)$$

where $\hat{H}$ is the one-electron Hamiltonian in the absence of disorder. Both the Green's function $\hat{G}$ and the self-energy $\hat{\Sigma}$ for electron scattering off disorder are matrices in the Keldysh space,

$$\hat{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} \hat{\Sigma}^R & \hat{\Sigma}^K \\ 0 & \hat{\Sigma}^A \end{pmatrix}. \quad (3.2)$$

Matrix $\hat{1}$ stands for unit matrix in the Keldysh space as well as in the one-electron Hilbert space.

Below we assume that the disorder potential is a combination of the long-range potential with correlation length ξ created by remote positively charged donors and the short-range potential with much smaller correlation length. When conditions $\xi \ll \lambda_H$ and $p_F l \gg 1$ are satisfied, the self-consistent Born approximation for the self-energy calculation is applicable.[36] In this case, the expression for the self-energy takes the form[17]

$$\hat{\Sigma}(\hat{\boldsymbol{p}}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} W(|\boldsymbol{q}|)[e^{i\boldsymbol{q}\hat{\boldsymbol{r}}}\hat{G}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\hat{\boldsymbol{r}}}]. \quad (3.3)$$

Here the function $W(|\boldsymbol{q}|)$ is introduced as a Fourier transform of the correlation function of a Gaussian disorder potential

$$\langle U(\boldsymbol{r}_1)U(\boldsymbol{r}_2)\rangle = \int \frac{d^2q}{(2\pi)^2} W(q) e^{i\boldsymbol{q}(\boldsymbol{r}_1-\boldsymbol{r}_2)}. \quad (3.4)$$

The external electric field can be eliminated by transferring to the moving reference frame, $\boldsymbol{r}\to\boldsymbol{r}+\zeta(t)$, where $\zeta(t)$ describes a two-dimensional electron motion in crossed electric and magnetic fields,

$$\partial_t\zeta(t) = \left(\frac{\partial_t - \omega_c\hat{\epsilon}}{\partial_t^2+\omega_c^2}\right)\frac{e\boldsymbol{E}(t)}{m}. \quad (3.5)$$

Here $\omega_c = |e|B/mc$ is the cyclotron frequency and $\hat{\epsilon}$ is the antisymmetric tensor: $\hat{\epsilon}\boldsymbol{E} = \boldsymbol{E}\times\boldsymbol{e}_z$ for any vector $\boldsymbol{E}$ lying in the x-y plane of the 2DES (see Fig. 1). In the moving reference frame, the disorder potential acting on electrons is time dependent, and Eq. (3.3) becomes

$$\hat{\Sigma}(\hat{\boldsymbol{p}}) = \int \frac{d^2q}{(2\pi)^2} W_{t_1t_2}(\boldsymbol{q})[e^{i\boldsymbol{q}\hat{\boldsymbol{r}}}\hat{G}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\hat{\boldsymbol{r}}}], \quad (3.6)$$

where

$$W_{t_1t_2}(\boldsymbol{q}) = W(q)e^{i\boldsymbol{q}\zeta_{t_1t_2}}, \quad (3.7a)$$

$$\zeta_{t_1t_2} = \zeta(t_1) - \zeta(t_2). \quad (3.7b)$$

To proceed further we introduce the operator of the guiding center coordinate $\hat{\boldsymbol{R}}$,

$$\hat{H} = \frac{\hat{\boldsymbol{p}}^2}{2m} - \mu, \quad \hat{\boldsymbol{r}} = \hat{\boldsymbol{R}} + \lambda_H^2\hat{\epsilon}\hat{\boldsymbol{p}}, \quad (3.8)$$

where $\lambda_H = (c\hbar/|e|B)^{1/2}$ is the magnetic length. The commutation relations between the operators of the guiding center $\hat{\boldsymbol{R}}$ and momentum $\hat{\boldsymbol{p}}$ are

$$[\hat{R}_\alpha, \hat{R}_\beta] = i\lambda_H^2\varepsilon_{\alpha\beta}, \quad [\hat{p}_\alpha,\hat{p}_\beta] = -\frac{i}{\lambda_H^2}\varepsilon_{\alpha\beta}, \quad [\hat{\boldsymbol{R}},\hat{\boldsymbol{p}}]=0. \quad (3.9)$$

Equation (3.6) takes the form

$$\hat{\Sigma}(\hat{\boldsymbol{p}}) = \int \frac{d^2q}{(2\pi)^2} W_{12}(\boldsymbol{q})[e^{i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}\lambda_H^2}\hat{G}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}\lambda_H^2}]. \quad (3.10)$$

We first analyze the retarded and advanced components of the Dyson equation, Eq. (3.1), which determine electron spectrum. Then, we reduce the equation for the Keldysh component of Eq. (3.1) to the kinetic equation for the electron distribution function.

A. Electron spectrum

The Dyson equation, Eq. (3.1), for the retarded component of the Green's function is given by

$$[i\partial_t - \hat{H}]\hat{G}^R(t,t_1;\hat{\boldsymbol{p}}) = \frac{\delta(t-t_1)}{2\pi} + \int_{t_1}^{t} dt_2\hat{\Sigma}^R(t,t_2)\hat{G}^R(t_2,t_1;\hat{\boldsymbol{p}}) \quad (3.11)$$

along with the self-consistency Eq. (3.10). We limit the analysis to the first order in the parameter $\lambda = e^{-\pi/\omega_c\tau_q}$. The zeroth-order solution in λ corresponds to the solution in the absence of a magnetic field. The standard answer for the self-energy in this case is

$$\hat{\Sigma}_0^R(t,t') = -\frac{i}{2\tau_q}\hat{I}_e\delta(t-t'), \quad (3.12)$$

where $\hat{I}_e$ is the unity operator in the coordinate space.

The Green's function corresponding to Eq. (3.12) is obtained by solving Eq. (3.11),

$$\hat{G}_0^R(t,t') = -i\theta(t-t')e^{-i\hat{H}(t-t')}e^{-(t-t')/2\tau_q}. \quad (3.13)$$

We consider the first iteration of the self-consistent Born approximation. For this purpose, we substitute Eq. (3.13) to the retarded matrix element of Eq. (3.10),

$$\hat{\Sigma}'^R_{tt'}(\boldsymbol{p}) = -i\theta(t-t')e^{-(t-t')/2\tau_q} \times \int \frac{d^2q}{(2\pi)^2} W_{tt'}(\boldsymbol{q}) e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}e^{-i\hat{H}(t-t')}e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}. \quad (3.14)$$

We notice that the operator product in the integrand in Eq. (3.14) can be written as

$$e^{i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}\lambda_H^2}e^{-i\hat{H}t}e^{-i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}\lambda_H^2} = e^{-i\hat{H}t}e^{i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}_t\lambda_H^2}e^{-i\boldsymbol{q}\hat{\epsilon}\hat{\boldsymbol{p}}\lambda_H^2}, \quad (3.15)$$

where the time-dependent operators are $\hat{\boldsymbol{p}}_t = e^{i\hat{H}t}\hat{\boldsymbol{p}}e^{-i\hat{H}t}$. To find the time dependence of the operators, it is convenient to introduce[37] "raising," $\hat{p}^+$, and "lowering," $\hat{p}^-$, operators, defined as $\hat{p}^{\pm}=(p_x \pm ip_y)/\sqrt{2}$ and obeying the commutation relation $[\hat{p}^+,\hat{p}^-]=-1/\lambda_H^2$. The introduced notations for $\hat{p}^{\pm}$ are useful as new operators have simple form in the interaction picture

$$\hat{p}_t^{\pm} = e^{\pm i\omega_c t}\hat{p}^{\pm}. \tag{3.16}$$

We also introduce the notations for vectors $q^{\pm}=(q_x \pm iq_y)/\sqrt{2}$ and write $\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}=q^-\hat{p}^+-q^+\hat{p}^-$. Using Eq. (3.16) and the operator relation $e^{\hat{A}}e^{\hat{B}}=e^{\hat{A}+\hat{B}}e^{(1/2)[\hat{A},\hat{B}]}$, we further transform Eq. (3.15) to

$$e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}_t\lambda_H^2}e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2} = \exp\left\{2i\lambda_H^2(\tilde{q}_t^-\hat{p}^+ + \tilde{q}_t^+\hat{p}^-)\sin\left[\frac{\omega_c t}{2}\right]\right\} \times\exp\left\{-i\lambda_H^2\frac{q^2}{2}\sin[\omega_c t]\right\}. \tag{3.17}$$

In Eq. (3.17), we denoted $\tilde{q}_t^{\pm}=q^{\pm}e^{\pm i\omega_c t/2}$, or in other words, the vector $\tilde{\boldsymbol{q}}_t$ is obtained from the vector $\boldsymbol{q}$ by a rotation on angle $\omega_c t/2$. Using Eqs. (3.15) and (3.17) we can write Eq. (3.14) in the following form:

$$\hat{\Sigma}_{tt'}^{\prime R}(\boldsymbol{p}) = -i\theta(t-t')e^{-(i\hat{H}+1/2\tau_q)(t-t')}\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}W_{tt'}(|\boldsymbol{q}|) \times\exp\left\{2i\lambda_H^2\tilde{\boldsymbol{q}}_{t-t'}\hat{\boldsymbol{p}}\,\sin\left[\frac{\omega_c}{2}(t-t')\right]\right\} \times\exp\left\{-i\lambda_H^2\frac{q^2}{2}\sin[\omega_c(t-t')]\right\}. \tag{3.18}$$

It is illustrative to consider the zero-field limit. In this case the momentum operator becomes a c number and Eq. (3.18) yields

$$\hat{\Sigma}_{tt'}^{\prime R}(\boldsymbol{p}) = -i\theta(t-t')e^{-(t-t')/2\tau_q}\int\frac{d^2q}{(2\pi)^2}W(q)e^{-i\xi_{\boldsymbol{p}-\boldsymbol{q}}(t-t')}. \tag{3.19}$$

Changing integration variables in Eq. (3.19) to $\xi_{\boldsymbol{p}-\boldsymbol{q}}$ and the angle formed by the vector $\boldsymbol{p}-\boldsymbol{q}$ with some fixed direction, we show the consistency of the employed approximation, namely,

$$\hat{\Sigma}^{\prime R}(\boldsymbol{p}) = \hat{\Sigma}_0^R \tag{3.20}$$

with $\hat{\Sigma}_0^R$ defined by Eq. (3.12) and

$$\frac{1}{\tau_q} = \int\frac{d\theta}{2\pi}\frac{1}{\tau_\theta}. \tag{3.21}$$

The scattering rate

$$\frac{1}{\tau_\theta} = 2\nu_0\pi W\left(2p_F\sin\frac{\theta}{2}\right) \tag{3.22}$$

off disorder on angle θ can be written in terms of its angular harmonics

$$\frac{1}{\tau_\theta} = \sum_{n=-\infty}^{+\infty}\frac{e^{in\theta}}{\tau_n}, \quad \tau_{-n}=\tau_n. \tag{3.23}$$

Then, $\tau_q=\tau_0$.

In finite magnetic fields, we notice that the exponential factors in Eq. (3.18) are $2\pi/\omega_c$ periodic. The argument of these exponents vanishes at $t-t'=lT_c$ with integer l and the integral in Eq. (3.18) diverges; $T_c=2\pi/\omega_c$ is the cyclotron period. We argue that this integral gives, in fact, a δ peak of the width $\delta t\approx 1/E_F$ for time difference $t-t'=lT_c$. Indeed, this statement is obvious if the operators in the exponent of Eq. (3.18) can be treated as commuting. For time intervals $|t-t'-lT_c|\lesssim 1/\sqrt{E_F\omega_c}$ the commutator of the two operators in the exponent of Eq. (3.18) is small since, in this case, each operator is multiplied by $\sin[\omega_c(t-t')/2]\lesssim\sqrt{\omega_c/E_F}$, and we can apply the same argument as the one used in zero magnetic field. On the other hand, for $1/\sqrt{E_F\omega_c}\lesssim|t-t'-lT_c|\leq T_c/2$ the result of the integration in Eq. (3.18) vanishes because of the rapid oscillations of the exponent. This can be checked explicitly by calculation of the matrix elements of the self-energy Eq. (3.18). We conclude that the noncommutativity of the operators can be ignored in Eq. (3.18), and we can apply our zero-field considerations whenever $t-t'$ is a multiple of the cyclotron period T_c.

The contributions to self-energy (3.14) with $l>1$ are proportional to λ^l and, therefore, can be neglected in moderately weak magnetic fields, when $\lambda\ll 1$. We stress that the terms with $l>1$ are beyond the accuracy of the first iteration of the self-consistent scheme. The corrections of the higher orders in λ can be taken into account in the spirit of Ref. 17, where the small scattering angle was only considered. In this paper, we restrict our analysis to terms $l=0,1$, sufficient in not too strong magnetic fields.

Introducing a new variable $\boldsymbol{p}'=\boldsymbol{p}-\boldsymbol{q}$ in Eq. (3.18) and neglecting the variation of the function $W_{t,t'}(\boldsymbol{p}'-\boldsymbol{p})$ given by Eq. (3.7) (see below), we can easily perform the integration over the absolute value $|\boldsymbol{p}'|$. This integration results in

$$\hat{\Sigma}_{t,t';\varphi}^{\prime R} = \frac{1}{i}\left(\frac{1}{2}\delta(t-t')-\lambda\delta(t-t'-T_c)\right)\hat{\mathcal{K}}_{t,t';\varphi}\{1\}, \tag{3.24}$$

where we have defined the integral kernel

$$\hat{\mathcal{K}}_{t,t';\varphi}\{F(\varphi)\} = \int\frac{d\varphi'}{2\pi}\frac{e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta_{t,t'}}}{\tau_{\varphi-\varphi'}}F(\varphi') \tag{3.25}$$

with an arbitrary function $F(\varphi)$; $\boldsymbol{n}_\varphi=(\cos\varphi;\sin\varphi;0)$. The negative sign of the second term in Eq. (3.24) corresponds to the chemical potential $\mu=s\omega_c$, with integer s, such that the density of states $\nu(\mu)$ is at minimum. We note that the exact position of the chemical potential is of no importance for our final results.

Equation (3.24) is valid if the variation in the matrix element for the transitions in the moving reference frame, $W_{t,t'}(\boldsymbol{p}'-\boldsymbol{p})$, can be neglected in the course of integration over the absolute value $|\boldsymbol{p}'|$. This requirement leads to the limitation on the strength of electric fields. Since the relevant time scale is $|t-t'-lT_c|\lesssim\tau_q\ll T_c$, which insures that the parameter λ can be defined in Eq. (3.24) unambiguously, the important range of the integration is $|p'|<1/v_F\tau_q$. The requirement of smoothness of $W_{t,t'}(\boldsymbol{p}'-\boldsymbol{p})$ in the momentum variable means that the distance electron drifts over one cyclotron period must be smaller than the quantum length $v_F\tau_q$. In the case of a constant electric field, this condition amounts to $v_D/\omega_c \ll v_F\tau_q$, or equivalently, $\epsilon_{dc}\ll E_F\tau_q$. If this condition is not met, the amplitude of oscillations as a function of ϵ_{dc} is reduced because of the finite broadening in time of the self-energy. In experiments,[21,27] $E_F\tau_q\approx 10^2$, $\epsilon_{dc}\lesssim 5$, and the above condition is satisfied. However, the amplitude of oscillations in Ref. 21 shows tendency to decrease when ε_{dc} increases. In our model, this tendency can be accounted for by taking into consideration the dependence of the function $W_{t,t'}(\boldsymbol{p}'-\boldsymbol{p})$ on the absolute value of momenta, which becomes stronger as ϵ_{dc} increases.

B. Electron distribution function

In Sec. III A, we analyzed the effect of disorder on the spectral characteristics of the electron Green's function, determined by the retarded and advanced components of the Green's function. Now we reduce the equation for the Keldysh component of the Green's function to the kinetic equation for the electron distribution function $\hat{f}$, related to the Keldysh component $\hat{G}^K$ through the standard expression

$$\hat{G}^K = \hat{G}^R - \hat{G}^A - 2[\hat{G}^R\hat{f} - \hat{f}\hat{G}^A]. \tag{3.26}$$

In the present analysis, the Wigner transformation $f(t,t';\boldsymbol{R},\boldsymbol{p})$ of the distribution function $\hat{f}$ in time and coordinate variables is independent of the "center-of-mass" coordinate $\boldsymbol{R}=(\boldsymbol{r}+\boldsymbol{r}')/2$ due to the translational symmetry. The peaked structure of the retarded and advanced Green's functions in Eq. (3.26) makes $f(t,\epsilon;\boldsymbol{R},\boldsymbol{p})$ to be independent of the absolute value of the momentum, $\boldsymbol{p}$. The dependence on the direction of the momentum is still to be retained. Notice that although the two components of the momentum operator are not commuting, the momentum direction is well defined in the quasiclassical regime $E_F\gg\omega_c$. Therefore, we can write $f(t,t';\boldsymbol{R},\boldsymbol{p})=f(t,t';\boldsymbol{n}_\varphi)=f_{t,t';\varphi}$. The solution of the resulting quantum kinetic equation presented in Sec. IV is consistent with the assumptions made above.

The distribution function $f(t,t';\boldsymbol{n}_\varphi)$ obeys the following kinetic equation:

$$[(\partial_t + i\hat{H});\hat{f}] = \mathrm{St}_{dis}\{\hat{f}\} + \mathrm{St}_{ee}\{\hat{f}\}, \tag{3.27}$$

where the notation $[\cdot;\cdot]$ stands for the commutator in one-particle Hilbert space and the time variable. In Eq. (3.27) the collision integral $\mathrm{St}_{ee}\{f\}$ describes the electron-electron interaction and is discussed in the end of this section. The collision integral $\hat{\mathrm{St}}_{dis}\{\hat{f}\}$ represents scattering off disorder and can be written as the sum of scattering "out" and "in" terms

$$\mathrm{St}_{dis} = \mathrm{St}_{out} + \mathrm{St}_{in}, \tag{3.28}$$

where

$$i\mathrm{St}_{out}\{\hat{f}\} = [\hat{\Sigma}^R\hat{f} - \hat{f}\hat{\Sigma}^A], \tag{3.29a}$$

$$i\mathrm{St}_{in}\{\hat{f}\} = \frac{1}{2}[\hat{\Sigma}^K - \hat{\Sigma}^R + \hat{\Sigma}^A]. \tag{3.29b}$$

Next, we analyze the collision integral in Eq. (3.27). We start our analysis with the scattering-out term, Eq. (3.29a), written as

$$i\mathrm{St}_{out}\{\hat{f}\}_{t,t';\varphi} = \int dt''[\hat{\Sigma}'^R_{t,t'';\varphi}f_{t'',t';\varphi} - f_{t,t'';\varphi}\hat{\Sigma}'^A_{t'',t';\varphi}]. \tag{3.30}$$

Using Eq. (3.24) we can perform integration over intermediate time t'' in Eq. (3.30) and obtain

$$\mathrm{St}_{out}\{\hat{f}\}_{t,t';\varphi} = -\hat{\mathcal{K}}_{t,t;\varphi}\{1\}f_{t,t';\varphi} + \lambda\hat{\mathcal{K}}_{t,t-T_c;\varphi}\{1\}f_{t-T_c,t';\varphi} + \lambda f_{t,t'-T_c;\varphi}\hat{\mathcal{K}}_{t'-T_c,t';\varphi}\{1\}, \tag{3.31}$$

where the operator $\hat{\mathcal{K}}_{t,t';\varphi}\{1\}$ acts on a unity as defined by Eq. (3.25) and $T_c=2\pi/\omega_c$.

We now turn to the consideration of the scattering-in term, Eq. (3.29b). We express the self-energies through the Green's functions using the self-consistency condition, Eq. (3.6), and the parametrization in Eq. (3.10),

$$\mathrm{St}_{in}\{\hat{f}\}_{t,t'} = i\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}W_{t,t'}(\boldsymbol{q})e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}[\hat{G}^R\hat{f} - \hat{f}\hat{G}^A]e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}, \tag{3.32}$$

where the Green's functions are given by Eq. (3.13). Commuting the retarded (advanced) Green's function with the left (right) exponent, we rewrite Eq. (3.32) as

$$\mathrm{St}_{in}\{\hat{f}\}_{t,t'} = i\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\int dt''W_{t,t'}(\boldsymbol{q}) \times[\hat{G}^R_{t,t''}e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}_{t-t''}\lambda_H^2}f_{t'',t'}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2} - e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}f_{t,t''}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}_{t'-t''}\lambda_H^2}\hat{G}^A_{t'',t'}]. \tag{3.33}$$

Using $e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}f_{t,t'}(\hat{\boldsymbol{p}})e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}=f_{t,t'}(\hat{\boldsymbol{p}}-\boldsymbol{q})$, we commute the exponents with the distribution function,

$$\mathrm{St}_{in}\{\hat{f}\}_{t,t'} = i\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\int dt''W_{t,t'}(\boldsymbol{q}) \times[\hat{G}^R_{t,t''}e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}_{t-t''}\lambda_H^2}e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}f_{t'',t'}(\hat{\boldsymbol{p}}-\boldsymbol{q}) - f_{t,t''}(\hat{\boldsymbol{p}}-\boldsymbol{q})e^{i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}\lambda_H^2}e^{-i\boldsymbol{q}\varepsilon\hat{\boldsymbol{p}}_{t'-t''}\lambda_H^2}\hat{G}^A_{t'',t'}]. \tag{3.34}$$

Following the same line of arguments as in the derivation of Eq. (3.24), we put Eq. (3.34) into the form

$$\mathrm{St}_{in}\{\hat{f}\}_{t,t';\varphi} = \int dt''\left\{\left(\frac{1}{2}\delta(t-t'') - \lambda\delta(t-t''-T_c)\right) \times\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t'',t';\varphi}\} + \left(\frac{1}{2}\delta(t'-t'') - \lambda\delta(t'-t''-T_c)\right)\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t,t'';\varphi}\}\right\}, \tag{3.35}$$

where $\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t_1,t_2;\varphi}\}$ is defined by Eq. (3.25). Performing the time integration in Eq. (3.35) we find

$$\mathrm{St}_{\mathrm{in}}\{\hat{f}\}_{t,t';\varphi} = \hat{\mathcal{K}}_{t,t';\varphi}\{f_{t,t';\varphi}\} - \lambda\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t-T_c,t';\varphi}\} - \lambda\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t,t'-T_c;\varphi}\}. \quad (3.36)$$

Collecting Eqs. (3.31) and (3.36) and using $i[\hat{H};f_{t,t';\varphi}] = \omega_c\partial_\varphi f_{t,t';\varphi}$ we can finally write down the kinetic equation

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial t'} + \omega_c\frac{\partial}{\partial\varphi}\right] f_{t,t';\varphi} = \mathrm{St}_{\mathrm{dis}}\{f\}_{t,t';\varphi} + \mathrm{St}_{\mathrm{ee}}\{f\}_{t,t';\varphi} \quad (3.37)$$

with the collision integral

$$\mathrm{St}_{\mathrm{dis}}\{f\}_{t,t';\varphi} = \hat{\mathcal{K}}_{t,t';\varphi}\{f_{t,t';\varphi}\} - \hat{\mathcal{K}}_{t,t;\varphi}\{1\}f_{t,t';\varphi} - \lambda\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t-T_c,t';\varphi}\} + \lambda\hat{\mathcal{K}}_{t,t-T_c;\varphi}\{1\}f_{t-T_c,t';\varphi} - \lambda\hat{\mathcal{K}}_{t,t';\varphi}\{f_{t,t'-T_c;\varphi}\} + \lambda f_{t,t'-T_c;\varphi}\hat{\mathcal{K}}_{t'-T_c,t';\varphi}\{1\}. \quad (3.38)$$

The integral kernel $\hat{\mathcal{K}}_{t,t';\varphi}\{F(\varphi)\}$ defined by Eq. (3.25) is the generalization of the corresponding differential operator derived in Ref. 17 for small-angle scattering.

We briefly discuss the term in the kinetic equation, Eq. (3.37), representing the electron-electron interaction, $\mathrm{St}_{\mathrm{ee}}\{f\}$. As it was shown in Ref. 18, electric fields produce an isotropic nonequilibrium contribution to the distribution function, which can only be stabilized by the inelastic relaxation mechanisms. In weak electric fields, the important inelastic relaxation is due to the electron-electron scattering. To take it into account, we keep the corresponding collision integral Eq. (3.42), which can be written in the energy representation for a steady in time distribution function $f(\varepsilon)$,

$$\mathrm{St}_{\mathrm{ee}}\{f(\varepsilon)\} = \int d\varepsilon' \int d\Omega M(\Omega,\varepsilon,\varepsilon') \times[\tilde{f}(\varepsilon)f(\varepsilon_+)\tilde{f}(\varepsilon')f(\varepsilon'_-) - f(\varepsilon)\tilde{f}(\varepsilon_+)f(\varepsilon')\tilde{f}(\varepsilon'_-)], \quad (3.39)$$

where $\tilde{f}(\varepsilon)\equiv 1-f(\varepsilon)$, $\varepsilon_+=\varepsilon+\Omega$, $\varepsilon'_-=\varepsilon'-\Omega$, and $M(\Omega,\varepsilon,\varepsilon')$ describes the dependence of the matrix element of the screened Coulomb interaction on the transferred energy Ω and the electron energies ε and ε'. The kernel $M(\Omega,\varepsilon,\varepsilon')$ has been discussed in details in Ref. 18. Below we use the linearized version of Eq. (3.39),

$$\mathrm{St}_{\mathrm{ee}}\{f(\varepsilon)\} = -\frac{f(\varepsilon) - f_T(\varepsilon)}{\tau_{\mathrm{ee}}}. \quad (3.40)$$

Here, τ_{ee} is the inelastic relaxation time due to the electron-electron interaction,

$$\frac{1}{\tau_{\mathrm{ee}}} \propto \frac{T^2}{4\pi\varepsilon_F}\ln\frac{\kappa v_F}{\max\{T,\sqrt{\omega_c^3\tau_{\mathrm{tr}}}\}}, \quad (3.41)$$

which can be obtained as a projection of the linearized electron-electron collision integral on the oscillating harmonic of the distribution function [see Eq. (4.4a) below].

C. Bilinear response in microwave field

In this subsection we simplify the integral kernel $\hat{\mathcal{K}}_{t,t'}$ in the limit of weak microwave power, keeping only terms which are bilinear in the microwave electric field, i.e., linear in power $\mathcal{P}_\omega$, introduced in Eq. (2.6). In the presence of microwave radiation $\hat{\mathcal{K}}_{t,t'}$ has the oscillatory dependence on the time variable $(t+t')/2$. The distribution function in turn acquires nonstationary corrections oscillating with the microwave frequency ω. It follows from the form of the kinetic equation, Eq. (3.37), that those corrections are small in the parameter on the order of $1/\omega\tau_{\mathrm{tr}}$ in systems with considered here mixed disorder.[34] We, therefore, neglect their contribution to the distribution function and consider only the stationary component of the distribution function. The latter can be found from the following equation:

$$\omega_c\partial_\varphi f_{t-t'}(\varphi) = \overline{\mathrm{St}}_{\mathrm{dis}}\{f\}_{t-t'} + \mathrm{St}_{\mathrm{ee}}\{f\}, \quad (3.42)$$

where the symbol $\overline{(\cdots)}$ stands for time averaging over one period of the microwave oscillations and we included the collision term due to the electron-electron interaction. Due to the $2\pi/\omega$ periodicity of the collision integral, Eq. (3.38), the time average of the kernel $\hat{\mathcal{K}}$, Eq. (3.25), is given by the integral over one period of the microwave field.

We consider the response of a two-dimensional electron gas to the in-plane electric field

$$\boldsymbol{E}_{\mathrm{tot}} = \boldsymbol{E} + \boldsymbol{E}_{\mathrm{mw}}, \quad (3.43)$$

represented as a superposition of a constant electric field $\boldsymbol{E}$ and a microwave field

$$\boldsymbol{E}_{\mathrm{mw}} = E_\omega\,\mathrm{Re}[\boldsymbol{e}^{\pm}e^{-i\omega t}]. \quad (3.44)$$

Here we consider the case of the circular polarized microwave radiation. The case of arbitrary polarization of microwave radiation is relegated to a separate investigation.[34] In Eq. (3.44) we introduced the complex polarization vector $\boldsymbol{e}^{\pm}=(\boldsymbol{e}_x \pm i\boldsymbol{e}_y)/\sqrt{2}$ with the property $\hat{\boldsymbol{\epsilon}}\boldsymbol{e}^{\pm} = \pm i\boldsymbol{e}^{\pm}$. In the above equations, the upper (lower) sign corresponds to the right (left) polarization of the microwave field propagating in the magnetic field direction (see Fig. 1).

The displacement ζ_{t_1,t_2}, Eq. (3.7b), is found by solving Eq. (3.5) with the electric field specified by Eqs. (3.43) and (3.44). The linearity of Eq. (3.5) allows us to represent its solution as the sum of the displacements in constant and microwave fields,

$$\zeta_{t_1,t_2} = \zeta^{\mathrm{dc}}_{t_1-t_2} + \zeta^{\mathrm{ac},\pm}_{t_1,t_2}, \quad (3.45)$$

where

$$\zeta^{\mathrm{dc}}_{t_1-t_2} = \frac{e(t_2-t_1)\hat{\boldsymbol{\epsilon}}\boldsymbol{E}}{m\omega_c}, \quad (3.46)$$

and

$$\zeta^{\mathrm{ac},\pm}_{t_1,t_2} = \frac{2\sqrt{2\mathcal{P}_\omega}}{p_F}\sin\frac{\omega(t_2-t_1)}{2}\mathrm{Im}[\boldsymbol{e}_{\pm}e^{-i\omega(t_1+t_2)/2}]. \quad (3.47)$$

The dimensionless parameter $\mathcal{P}_\omega$ has been introduced in Eq. (2.6). We make an expansion of the integral kernel $\hat{\mathcal{K}}_{t,t'}$ in

Eq. (3.25) to the first order in $\mathcal{P}_\omega$ using the relation

$$e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta_{t_1,t_2}} \approx e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta^{\rm dc}_{t_1-t_2}}\left\{1-\frac{p_F^2}{2}[(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta^{\rm ac}_{t_1,t_2}]^2\right\}. \tag{3.48}$$

Averaging Eq. (3.48) with respect to time results in

$$\overline{[(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta^{\rm ac}_{t_1,t_2}]^2} = \frac{2\mathcal{P}_\omega}{p_F^2}(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})^2\sin^2\frac{\omega(t_1-t_2)}{2}. \tag{3.49}$$

Finally the collision kernel to the second order in the microwave field takes the form

$$\hat{\mathcal{K}}_{t,t';\varphi}\{F(\varphi)\} = \int \frac{d\varphi'}{2\pi}e^{iW_{\varphi\varphi'}(t-t')}F(\varphi') \times\left[\frac{1}{\tau_{\varphi'-\varphi}} - \mathcal{P}_\omega\frac{1-\cos\,\omega(t-t')}{\overline{\tau}_{\varphi'-\varphi}}\right], \tag{3.50}$$

where the rate $1/\overline{\tau}$ has been introduced in Eq. (2.5) and the quantity

$$W_{\varphi\varphi'} = eER_c(\sin\,\varphi - \sin\,\varphi') \tag{3.51}$$

is the work done by the dc electric field as the result of scattering off an impurity,[26] $W_{\varphi\varphi'}=e\boldsymbol{E}\Delta\boldsymbol{R}_{\varphi\to\varphi'}$ with the shift of the cyclotron orbit $\Delta\boldsymbol{R}_{\varphi\to\varphi'}$ given by Eq. (2.1).

The collision integral due to scattering off disorder is obtained from Eq. (3.50) by performing the Fourier transformation in time variable $t-t'$. We represent it as the sum of two terms describing two separate scattering mechanisms

$$\overline{\rm St}_{\rm dis} = {\rm St}_{\rm dc} + {\rm St}_{\rm mw}. \tag{3.52}$$

In Eq. (3.52) the first term corresponds to the scattering off the impurities in the absence of the microwave radiation[26]

$${\rm St}_{\rm dc}f = \int \frac{d\varphi'}{2\pi}\frac{\nu(\varepsilon+W_{\varphi\varphi'})}{\nu_0}\frac{f(\varepsilon+W_{\varphi\varphi'},\varphi')-f(\varepsilon,\varphi)}{\tau_{\varphi'-\varphi}}. \tag{3.53}$$

The second term in Eq. (3.52),

$${\rm St}_{\rm mw}f = -\frac{\mathcal{P}_\omega}{2\nu_0}\sum_{\pm}\int\frac{d\varphi'}{2\pi}\frac{1}{\overline{\tau}_{\varphi'-\varphi}}\{\nu(\varepsilon+W_{\varphi\varphi'}) \times[f(\varepsilon+W_{\varphi\varphi'},\varphi')-f(\varepsilon,\varphi)] - \nu(\varepsilon+W_{\varphi\varphi'}\pm\omega) \times[f(\varepsilon+W_{\varphi\varphi'}\pm\omega,\varphi')-f(\varepsilon,\varphi)]\}, \tag{3.54}$$

describes the scattering processes of electrons off impurities with participation of a microwave radiation quantum. For the analysis limited to the first order in the microwave power, all multiphoton processes are neglected. More specifically, the first term in Eq. (3.54) represents the impurity scattering with one photon emitted (absorbed) virtually and can be thought of as the renormalization of the impurity potential by microwave radiation. This term is taken into account in Sec. II A as a linear in $\mathcal{P}_\omega$ contribution to $j_{\rm el}$ in Eq. (2.2). The second term in Eq. (3.54) describes the real processes of emission (absorption) of one microwave quantum accompanying the impurity scattering. It is this term which produces the oscillatory ω/ω_c dependence of the magnetoresistance and corresponds to $j_{\rm in}$ in Eq. (2.2).

We notice that the obtained collision integral vanishes in the clean system for any frequency ω away from the cyclotron resonance. In the clean limit, the conductivity tensor can be found by applying the Kohn's theorem[38] argumentation. The conductivity tensor of an interacting system, which is galilean invariant, is identical to that of the noninteracting system.[37] It follows then that away from the cyclotron resonance, $\omega\neq\omega_c$, the electric field appears in the collision integral only in a combination with the disorder scattering rate $1/\tau_\theta$.

IV. MAGNETO-OSCILLATIONS IN THE PRESENCE OF ac AND dc EXCITATIONS

In this section we calculate and analyze the dissipative current

$$j = 2ev_F\int\frac{d\varphi}{2\pi}\cos\,\varphi\int\nu(\varepsilon)f(\varepsilon,\varphi)d\varepsilon, \tag{4.1}$$

where the distribution function is determined as a solution of kinetic equation (3.42) with the collision integral given by Eqs. (3.52)–(3.54).

A. Solution of the kinetic equation

We look for the solution of the kinetic equation, Eq. (3.42), in the form

$$f(\varepsilon,\varphi) = f_T(\varepsilon) + \delta f_{\rm cl}(\varepsilon,\varphi) + \delta f_0(\varepsilon) + \delta f_1(\varepsilon,\varphi), \tag{4.2}$$

where the first term is the equilibrium Fermi-Dirac distribution function, Eq. (2.10). The second term is the classical solution corresponding to the constant density of states

$$\delta f_{\rm cl}(\varepsilon,\varphi) = -\partial_\varepsilon f_T\frac{eER_c}{\omega_c\tau_{\rm tr}}\cos\,\varphi, \tag{4.3}$$

leading to the Drude result for the longitudinal conductivity at large Hall angle [Eq. (2.13)].

The third and fourth terms in Eq. (4.2) are the zeroth and first angular harmonics of the correction to the distribution function resulting from the quantum oscillatory component of the density of states in collision integral (3.52). For $\omega_c\tau_{\rm tr}\gg 1$, we keep only the isotropic component and the first angular harmonic of the distribution function,

$$\delta f_0(\varepsilon) = \lambda\partial_\varepsilon f_T I\,\sin\frac{2\pi\varepsilon}{\omega_c} \tag{4.4a}$$

$$\delta f_1(\varepsilon,\varphi) = \lambda\partial_\varepsilon f_T\left[A_1\cos\frac{2\pi\varepsilon}{\omega_c}+\lambda A_2\right]\cos\,\varphi. \tag{4.4b}$$

The coefficients I, A_1, and A_2 are fixed by kinetic equation (3.42). The calculation of these coefficients is outlined in the Appendix.

The amplitude I of the isotropic part of the distribution function is

$$I=-\frac{\omega_c}{\pi}\frac{1}{\tau_{ee}^{-1}+\tau_0^{-1}-\gamma(\epsilon_{dc})+2\mathcal{P}_\omega\bar{\gamma}(\epsilon_{dc})\sin^2\pi\epsilon_{ac}}[\epsilon_{dc}\gamma'(\epsilon_{dc}) - 2\pi\epsilon_{ac}\sin 2\pi\epsilon_{ac}\mathcal{P}_\omega\bar{\gamma}(\epsilon_{dc})-2\epsilon_{dc}\sin^2\pi\epsilon_{ac}\mathcal{P}_\omega\bar{\gamma}'(\epsilon_{dc})]. \quad (4.5)$$

Here, functions $\gamma(\epsilon_{dc})$ and $\bar{\gamma}(\epsilon_{dc})$ are defined as

$$\gamma(\epsilon_{dc})=\sum_n\frac{J_n^2(\pi\epsilon_{dc})}{\tau_n} \quad (4.6a)$$

and

$$\bar{\gamma}(\epsilon_{dc})=\sum_n J_n^2(\pi\epsilon_{dc})\left[\frac{1}{\tau_n}-\frac{1}{2\tau_{n+1}}-\frac{1}{2\tau_{n-1}}\right] \quad (4.6b)$$

in terms of the Bessel functions $J_n(\pi\epsilon_{dc})$ of the order n and angular harmonics $1/\tau_n$ of scattering rate off disorder [Eq. (3.23)]. Term τ_0^{-1} in the denominator of Eq. (4.5) is the $n=0$ harmonic of scattering rate off disorder and coincides with the quantum-scattering rate off disorder. Coefficient I contains the inelastic relaxation time τ_{ee} [Eq. (3.41)]. Equation (4.5) was obtained in Ref. 26 for arbitrary value of ϵ_{dc} and $\mathcal{P}_\omega=0$ and in Ref. 18 for $\epsilon_{dc}\ll 1$ and $\mathcal{P}_\omega \ll 1$, and has been shown to be important for the description of MIRO and HIRO at small voltages. Equation (4.5) determines the behavior of the isotropic nonequilibrium component of the distribution function in constant electric field of an arbitrary strength.

The amplitude A_1 is given by

$$A_1=-\frac{I}{\pi\omega_c}[\gamma'(\epsilon_{dc})-2\mathcal{P}_\omega\bar{\gamma}'(\epsilon_{dc})\sin^2\pi\epsilon_{ac}]+\frac{1}{\pi^2}[\epsilon_{dc}\gamma''(\epsilon_{dc}) - 2\pi\epsilon_{ac}\sin 2\pi\epsilon_{ac}\mathcal{P}_\omega\bar{\gamma}'(\epsilon_{dc})-2\epsilon_{dc}\sin^2\pi\epsilon_{ac}\mathcal{P}_\omega\bar{\gamma}''(\epsilon_{dc})]. \quad (4.7)$$

Finally we obtain the amplitude A_2 in the following form:

$$A_2=\frac{I}{\pi\omega_c}[\gamma'(\epsilon_{dc})-2\mathcal{P}_\omega\bar{\gamma}'(\epsilon_{dc})\sin^2\pi\epsilon_{ac}]. \quad (4.8)$$

Equations (4.5), (4.7), and (4.8) determine the first two angular harmonics of the distribution function through Eq. (4.4). This allows us to compute the current density as discussed in Sec. IV B.

B. Nonlinear current

In this section we calculate the dependence of the current to the first order in power $\mathcal{P}_\omega$ on parameters ϵ_{dc} and ϵ_{ac}, characterizing the strength of the dc and ac excitations, respectively. We substitute the distribution function Eq. (4.2) with factors I and $A_{1,2}$ given by Eqs. (4.5), (4.7), and (4.8) to the expression for the dissipative current, Eq. (4.1),

$$j(\epsilon_{dc},\epsilon_{ac})=\sigma_D E+\delta j, \quad (4.9)$$

where the Drude conductivity σ_D is given by Eq. (2.13) and the leading correction in λ^2 to the current has the form

$$\delta j=\lambda^2(A_1-A_2)ev_F\nu_0. \quad (4.10)$$

Substituting $A_{1,2}$ from Eqs. (4.7) and (4.8) to Eq. (4.10), we represent the correction to the current in terms of the dimensionless function $F(\epsilon_{dc},\epsilon_{ac})$,

$$\delta j=2\sigma_D E\lambda^2 F(\epsilon_{dc},\epsilon_{ac}). \quad (4.11)$$

The function $F(\epsilon_{dc},\epsilon_{ac})$ in Eq. (4.11) describes the leading correction to the classical value of the current due to oscillations of the electron density of states. This function can be represented as a combination of the displacement term $F_d(\epsilon_{dc},\epsilon_{ac})$ and the inelastic term $F_i(\epsilon_{dc},\epsilon_{ac})$, arising due to the nonequilibrium isotropic component of the distribution function,

$$F(\epsilon_{dc},\epsilon_{ac})=F_d(\epsilon_{dc},\epsilon_{ac})+F_i(\epsilon_{dc},\epsilon_{ac}). \quad (4.12)$$

The displacement contribution F_d originates from the modification of the scattering rates in crossed electric and magnetic fields, while neglecting the effect of electric fields on the isotropic component of the electron distribution function. We have

$$\frac{F_d}{\tau_{tr}}=-\frac{\gamma''(\epsilon_{dc})}{\pi^2}+\frac{2\mathcal{P}_\omega}{\pi^2}\left(\frac{\pi\epsilon_{ac}\bar{\gamma}'\sin 2\pi\epsilon_{ac}}{\epsilon_{dc}}+\bar{\gamma}''\sin^2\pi\epsilon_{ac}\right) \quad (4.13)$$

with $\gamma=\gamma(\epsilon_{dc})$ and $\bar{\gamma}=\bar{\gamma}(\epsilon_{dc})$ defined by Eqs. (4.6).

The remaining inelastic contribution is proportional to the amplitude I of the isotropic and energy-dependent nonequilibrium component of the distribution function $\delta f_0(\varepsilon)$. This contribution is sensitive to the energy relaxation rate $1/\tau_{ee}$ at sufficiently small values of ϵ_{dc},

$$\frac{F_i}{\tau_{tr}}=-\frac{2}{\pi^2\epsilon_{dc}}\frac{\gamma'-2\bar{\gamma}'\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}}{\tau_{ee}^{-1}+\tau_0^{-1}-\gamma+2\bar{\gamma}\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}} \times(\epsilon_{dc}\gamma'-2\pi\epsilon_{ac}\bar{\gamma}\mathcal{P}_\omega\sin 2\pi\epsilon_{ac}-2\epsilon_{dc}\bar{\gamma}'\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}). \quad (4.14)$$

Equations (4.11)–(4.14) give the explicit expression for the current in response to the applied dc electric field with strength $E=\epsilon_{dc}\omega_c/(2|e|R_c)$ in weak microwave fields $\mathcal{P}_\omega\ll 1$.

First, we analyze the properties of functions $F_d(\epsilon_{dc},\epsilon_{ac})$ and $F_i(\epsilon_{dc},\epsilon_{ac})$ in a weak dc electric field, $\epsilon_{dc}\ll 1$. In this case

$$\gamma(\epsilon_{dc})=\frac{1}{\tau_0}-\frac{\pi^2\epsilon_{dc}^2}{2\tau_{tr}}+\frac{\pi^4\epsilon_{dc}^4}{32}\frac{1}{\tau_*}, \quad (4.15a)$$

$$\bar{\gamma}(\epsilon_{dc})=\frac{1}{\tau_{tr}}-\frac{1}{4}\pi^2\epsilon_{dc}^2\frac{1}{\tau_*}, \quad (4.15b)$$

where $1/\tau_0\equiv 1/\tau_{n=0}$ is the quantum-scattering rate off disorder,

$$\frac{1}{\tau_{tr}}=\frac{1}{\tau_0}-\frac{1}{\tau_1} \quad (4.16)$$

is the transport scattering rate written in terms of harmonics of scattering rate [see Eqs. (2.13) and (3.23)], and

$$\frac{1}{\tau_*}=\frac{3}{\tau_0}-\frac{4}{\tau_1}+\frac{1}{\tau_2}. \tag{4.17}$$

Substituting Eqs. (4.15) to Eqs. (4.13) and (4.14), we obtain

$$F_d=1-\frac{\tau_{tr}}{\tau_*}\left[\frac{3}{8}\pi^2\epsilon_{dc}^2+\mathcal{P}_\omega(\pi\epsilon_{ac}\sin 2\pi\epsilon_{ac}+\sin^2\pi\epsilon_{ac})\right] \tag{4.18}$$

and

$$F_i=-2\frac{1-(\tau_{tr}/\tau_*)\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}}{\tau_{tr}/\tau_{ee}+\pi^2\epsilon_{dc}^2/2+2\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}} \times\{\pi^2\epsilon_{dc}^2[1-(\tau_{tr}/\tau_*)\mathcal{P}_\omega\sin^2\pi\epsilon_{ac}]+2\pi\mathcal{P}_\omega\epsilon_{ac}\sin 2\pi\epsilon_{ac}\}. \tag{4.19}$$

In smooth disorder, $\tau_{tr}\ll\tau_*$. In this case, Eq. (4.19) coincides with the contribution to the electric current, considered in Ref. 18, which we already presented in Eq. (2.20). The nonlinear dependence on the applied electric fields occur at $\epsilon_{dc}\lesssim\sqrt{\tau_{tr}/\tau_{ee}}$.

In strong dc electric-fields functions $\gamma(\epsilon_{dc})$ and $\bar{\gamma}(\epsilon_{dc})$ can be evaluated for $\epsilon_{dc}\gg 1$ using the asymptotes of the Bessel functions. As a result, we find the displacement contribution $F_d(\epsilon_{dc},\epsilon_{ac})$ in the form

$$\frac{F_d}{\tau_{tr}}=\frac{4}{\pi^2}\frac{1}{\epsilon_{dc}\tau_\pi}\sin 2\pi\epsilon_{dc}(1-2\mathcal{P}_\omega)+\frac{4}{\pi^2}\frac{\mathcal{P}_\omega}{\epsilon_{dc}\tau_\pi}\sum_\pm\frac{\epsilon_{dc}\pm\epsilon_{ac}}{\epsilon_{dc}}\sin 2\pi(\epsilon_{dc}\pm\epsilon_{ac}) \tag{4.20}$$

with $1/\tau_\pi=\Sigma_n\cos\pi n/\tau_n$ being the backscattering rate off disorder. The function $F_d(\epsilon_{dc},\epsilon_{ac})$ has an oscillatory dependence on the parameter ϵ_{dc}. We presented the corresponding expression for the nonlinear contribution to the current in Sec. II, Eqs. (2.15) and (2.16), where the same result was obtained from semiqualitative arguments and the stationary phase approximation.

For the inelastic contribution F_i in the limit $\epsilon_{dc}\gg 1$ we obtain the following estimate:

$$\frac{F_i}{\tau_{tr}}=-\frac{8}{\pi^4}\frac{1}{\epsilon_{dc}\tau_\pi}\frac{\tau_0}{\tau_\pi\epsilon_{dc}}\cos^2 2\pi\epsilon_{dc}\left[1-2\mathcal{P}_\omega\left(4\sin^2\pi\epsilon_{ac}+\frac{\epsilon_{ac}}{\epsilon_{dc}}\tan 2\pi\epsilon_{dc}\sin 2\pi\epsilon_{ac}\right)\right]. \tag{4.21}$$

We notice that this contribution $F_i(\epsilon_{dc},\epsilon_{ac})$ is smaller than $F_d(\epsilon_{dc},\epsilon_{ac})$ by factor $(1/\epsilon_{dc})$ in strong electric fields $\epsilon_{dc}\gg 1$. For a system with smooth disorder $F_i(\epsilon_{dc},\epsilon_{ac})$ contains also an additional small factor $\tau_0/\tau_\pi\ll 1$. This smallness allows one to neglect the inelastic contribution to the current in strong electric fields. The first line of Eq. (4.21) coincides[39] with the asymptote found in Ref. 26. The second line of Eq. (4.21) describes the effect of microwave field in the bilinear response.

C. Model of disorder

The purpose of this section is to analyze our results for a particular model of disorder. The disorder in GaAs/AlGaAs heterostructures is mainly due to remote charged donors. The potential created by these donors is smooth on the electron wavelength scale. Therefore, such potential is characterized by an exponentially suppressed backscattering amplitude. As we saw in Sec. II A the latter is crucial for the onset of nonlinear magneto-oscillations at higher current densities. On the other hand the relatively weak in-plane disorder creates the scattering potential of the short range, giving rise to a finite backscattering amplitude.

An adequate model should describe a mixed disorder that includes impurities with narrow and wide angle scattering.[40] Specifically, we use the expression for the angular harmonics of the scattering rate employed in Ref. 26,

$$\frac{1}{\tau_n}=\frac{\delta_{n,0}}{\tau_{sh}}+\frac{1}{\tau_{sm}}\frac{1}{1+\chi n^2}. \tag{4.22}$$

For this model we have the following expressions for the quantum and transport scattering rates:

$$\frac{1}{\tau_q}=\frac{1}{\tau_0}=\frac{1}{\tau_{sh}}+\frac{1}{\tau_{sm}},\qquad \frac{1}{\tau_{tr}}=\frac{1}{\tau_{sh}}+\frac{1}{\tau_{sm}}\frac{\chi}{1+\chi}. \tag{4.23}$$

We perform calculations under the assumptions $\omega_c\tau_q\lesssim 1$ and $\omega_c\tau_{tr}\gg 1$, which can be met for disorder described by Eq. (4.22), provided that $\chi\ll 1$ and $\tau_{sh}\gg\tau_{sm}$. The last inequality means that the long-range disorder is stronger than the short-range disorder. The scattering time τ_* is given by

$$\frac{1}{\tau_*}=\frac{3}{\tau_{sh}}+\frac{12\chi^2}{\tau_{sm}}. \tag{4.24}$$

Equations (4.23) and (4.24) determine the ratio τ_{tr}/τ_*, which appears in Eqs. (4.18) and (4.19) for the nonlinear contributions to the current at weak electric fields.

For very small values of ϵ_{dc} and $\mathcal{P}_\omega$, we have the following estimate:

$$F_i=-\frac{2\tau_{ee}}{\tau_{tr}}(\pi^2\epsilon_{dc}^2+2\pi\mathcal{P}_\omega\epsilon_{ac}\sin 2\pi\epsilon_{ac}). \tag{4.25}$$

Comparison of this expression with Eq. (4.18) shows that the linear in $\mathcal{P}_\omega$ and ϵ_{dc}^2 contributions to the nonlinear current are dominated by the inelastic mechanism if $\tau_{ee}/\tau_{tr}\gg\tau_{tr}/\tau_*$. If only long-range disorder is present in the system, we obtain for the ratio $\tau_{tr}/\tau_*=12\chi\ll 1$ and the inelastic mechanism is indeed dominant for sufficiently low temperatures, while $\tau_{ee}\gtrsim\tau_{tr}\chi\sim\tau_0$. At higher temperatures the interaction effects suppress the oscillations of the density of states and decrease the overall amplitude of nonlinear current. In the case of mixed disorder, parameter τ_{tr}/τ_* could be on the order of unity and the inelastic and displacement mechanisms become comparable at lower temperatures when $\tau_{ee}\sim\tau_{tr}$, but still $\tau_{ee}\gg\tau_0$, and the oscillations of the density of states are not smeared by the interaction effects. As a consequence, there exists a regime where the linear photoresistivity may have significantly weak dependence on electron temperature.[41]

We use the model for disorder described by Eq. (4.22) to evaluate the backscattering rate, which characterizes nonlinear contributions to the current in strong electric fields $\epsilon_{dc}\gg 1$. Performing summation over index n for $\theta=\pi$ in Eq. (3.23), we find

$$\frac{1}{\tau_\pi} = \frac{1}{\tau_{\rm sh}} + \frac{1}{\tau_{\rm sm}} \frac{2\pi}{\sqrt{\chi}} \exp\left(-\frac{\pi}{\sqrt{\chi}}\right). \tag{4.26}$$

This rate controls the magneto-oscillations for $\epsilon_{\rm dc} \gg 1$ [Eqs. (4.20) and (4.21)]. For small values of χ, the second term can be disregarded.

To analyze the case of arbitrary $\epsilon_{\rm dc}$, we evaluate functions $\gamma(\epsilon_{\rm dc})$ and $\bar{\gamma}(\epsilon_{\rm dc})$, Eqs. (4.6), in the limit $\chi \ll 1$. We perform summation over index n in Eq. (4.6) using the Poisson formula and neglect exponentially small corrections, similar to the second term in Eq. (4.26). The result reads[26]

$$\gamma(\epsilon_{\rm dc}) = \frac{J_0^2(\pi\zeta)}{\tau_{\rm sh}} + \frac{1}{\tau_{\rm sm}} \frac{1}{\sqrt{1+\chi\pi^2\epsilon_{\rm dc}^2}}, \tag{4.27a}$$

$$\bar{\gamma}(\epsilon_{\rm dc}) = \frac{1}{\tau_{\rm sh}}[J_0^2(\pi\epsilon_{\rm dc}) - J_1^2(\pi\epsilon_{\rm dc})] + \frac{\chi}{\tau_{\rm sm}} \frac{1-\chi\pi^2\epsilon_{\rm dc}^2/2}{(1+\chi\pi^2\epsilon_{\rm dc}^2)^{5/2}}. \tag{4.27b}$$

Equation (4.27) is convenient for calculating the nonlinear contribution to the current at arbitrary value of $\epsilon_{\rm dc}$. The resulting function $F(\epsilon_{\rm dc}, \epsilon_{\rm ac})$, Eq. (4.12), is plotted in Figs. 2(a) and 2(b). We also compare contributions $F_d(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ and $F_i(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ to the net nonlinear current. Both functions are shown for the same set of parameters in Fig. 2(c). The inelastic contribution $F_i(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ has a large variation at small $\epsilon_{\rm dc}$, and vanishes at larger values of $\epsilon_{\rm dc}$. On the other hand, the displacement component $F_d(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ is an oscillatory function of the parameter $\epsilon_{\rm dc}$ with only weakly decaying amplitude.

For $\epsilon_{\rm dc} \gtrsim 1$, when the inelastic contribution becomes small and the function $F(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ coincides with the displacement contribution $F_d(\epsilon_{\rm dc}, \epsilon_{\rm ac})$. The latter is a sum of two terms. One term contains $1/\tau_{\rm sm}$, this term varies smoothly as a function of $\epsilon_{\rm dc}$ on the scale $\epsilon_{\rm dc} \sim 1/\sqrt{\chi}$. Another term contains $1/\tau_{\rm sh}$ and oscillates as a function of $\epsilon_{\rm dc}$ with period equal to unity. The corresponding oscillating term can be written as

$$\frac{F_{d,\rm osc}}{\tau_{\rm tr}} = -\frac{[J_0^2(\pi\epsilon_{\rm dc})]''}{\pi^2\tau_{\rm sh}} + \frac{2\mathcal{P}_\omega}{\pi^2\tau_{\rm sh}}\Bigg\{[J_0^2(\pi\epsilon_{\rm dc}) - J_1^2(\pi\epsilon_{\rm dc})]'' \times \sin^2 \pi\epsilon_{\rm ac} + \frac{\pi\epsilon_{\rm ac}}{\epsilon_{\rm dc}}[J_0^2(\pi\epsilon_{\rm dc}) - J_1^2(\pi\epsilon_{\rm dc})]' \sin 2\pi\epsilon_{\rm ac}\Bigg\}. \tag{4.28}$$

This equation indicates clearly the vital importance of the short-range disorder, characterized by the finite backscattering rate $1/\tau_{\rm sh}$ for the onset of the magneto-oscillations in the nonlinear transport regime.

D. Differential magnetoresistance

Now we apply Eqs. (4.9) and (4.11) for the electric current response to the applied dc electric field to describe the longitudinal differential magnetoresistance

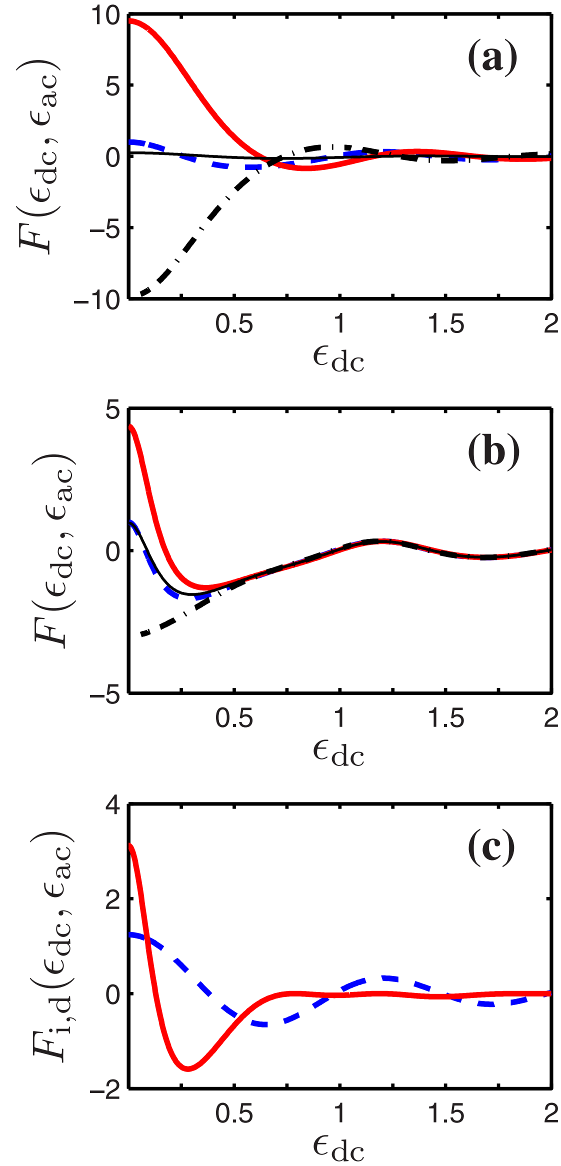

FIG. 2. (Color online) The function $F(\epsilon_{\rm dc}, \epsilon_{\rm ac})$ as defined by Eqs. (4.12)–(4.14) with $\gamma(\epsilon_{\rm dc})$ and $\bar{\gamma}(\epsilon_{\rm dc})$ given by Eq. (4.26) for the model of disorder introduced in Sec. IV C. (a) $\tau_{\rm sm}=1$, $\tau_{\rm sh}=1$, $\tau_{\rm ee}=0.5$, $\chi=0.0001$, and $\mathcal{P}_\omega=0.25$; (b) $\tau_{\rm sm}=0.1$, $\tau_{\rm sh}=1$, $\tau_{\rm ee}=10$, $\chi=0.001$, and $\mathcal{P}_\omega=0.01$. In both cases the thick solid (red) line, the dashed (blue) line, the dashed-dotted (black) line, and the thin solid (black) line are used for $\epsilon_{\rm ac}=2.75$, $\epsilon_{\rm ac}=3$, $\epsilon_{\rm ac}=3.25$, and $\epsilon_{\rm ac}=3.5$, respectively. Panel (c) shows the inelastic contribution F_i, solid (red) line, and displacement contribution F_d, dashed (blue) line for the set of parameters of panel (b) at $\epsilon_{\rm ac}=2.75$.

$$\rho(j) = \partial E_\parallel / \partial j. \tag{4.29}$$

In Eq. (4.29) we have chosen $\boldsymbol{e}_x$ to be the current direction, so that $E_\parallel$ is the electric-field component parallel to the current. In the limit $\omega_c \gg 1/\tau_{\rm tr}$ we write

$$E_\parallel = \rho_D j, \quad \rho_D = \frac{1}{e^2\nu_0 v_F^2 \tau_{\rm tr}}, \tag{4.30}$$

where the electric current density j is given by Eq. (4.9)

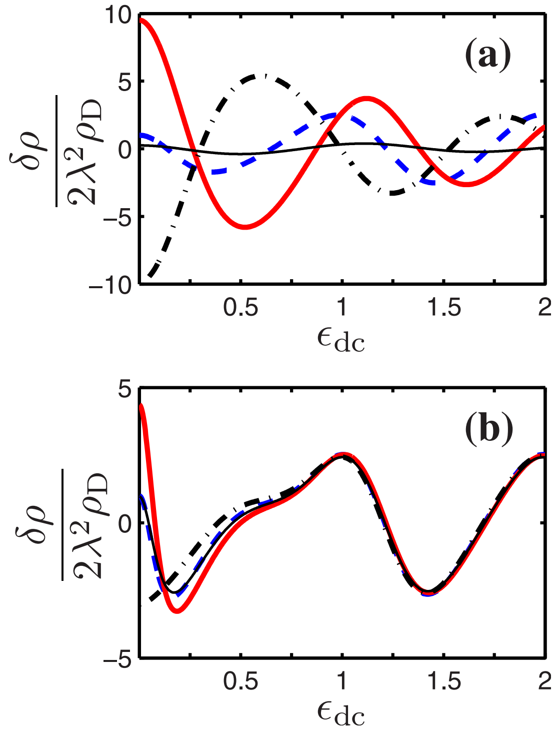

FIG. 3. (Color online) The upper and lower panels show the differential resistivity as a function of ϵ_{dc} for values of parameters used in Figs. 2(a) and 2(b), respectively. The differential resistance is obtained by substitution of $F(\epsilon_{dc},\epsilon_{ac})$, shown in Figs. 2(a) and 2(b), to Eq. (4.33).

$$E_{\parallel} = \rho_D j[1 + 2\lambda^2 F(\epsilon^j_{dc},\epsilon_{ac})], \tag{4.31}$$

calculated with a total electric field $E \simeq \rho_H j$, where $\rho_H = \omega_c/(e^2 v_F^2 \nu_0)$ is the Hall resistance and

$$\epsilon^j_{dc} = \frac{2|e|(\rho_H j)R_c}{\omega_c} = \frac{4\pi j}{e p_F \omega_c}. \tag{4.32}$$

We finally obtain for the oscillatory part of the differential resistance, $\delta\rho = \rho - \rho_D$ the following expression:

$$\frac{\delta\rho(j)}{\rho_D} = 2\lambda^2 \frac{d}{d\epsilon^j_{dc}}[\epsilon^j_{dc} F(\epsilon^j_{dc},\epsilon_{ac})] \tag{4.33}$$

with function $F(\epsilon^j_{dc},\epsilon_{ac})$ given by Eqs. (4.12)–(4.14). Figure 3 shows the differential resistance found for the specific disorder model considered in Sec. IV C.

We analyze our results in the limiting cases of small and large dc currents. We start with the discussion of the differential resistance $\delta\rho$ at small direct current, $\epsilon^j_{dc} \lesssim 1$, when in samples with smooth disorder the nonlinear behavior originates from $F_i(\epsilon^j_{dc},\epsilon_{ac})$ contribution. Keeping only this contribution in Eq. (4.33), at $\epsilon^j_{dc}=0$ we have

$$\frac{\delta\rho}{\rho_D} = -(2\lambda)^2 \frac{2\pi\epsilon_{ac}\mathcal{P}_\omega \sin 2\pi\epsilon_{ac}}{\tau_{tr}/\tau_{ee} + 2\mathcal{P}_\omega \sin^2 \pi\epsilon_{ac}}. \tag{4.34}$$

As ϵ^j_{dc} increases, the differential resistance decreases from the above value on the scale $\epsilon^j_{dc} \simeq \sqrt{\tau_{tr}/\tau_{ee}}$ and exhibits a nonmonotonic behavior at $\epsilon^j_{dc} \gtrsim 1$. However for large values of ϵ^j_{dc}, the inelastic contribution to the differential resistance vanishes fast, as discussed in Sec. IV B.

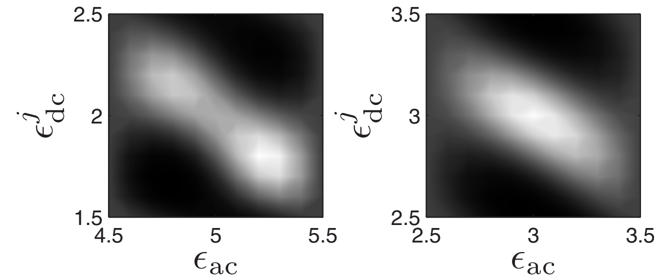

FIG. 4. The differential resistance obtained from Eq. (4.35) for $\mathcal{P}_\omega=0.25$ presented in a grayscale plot. The bright areas correspond to a higher value of the resistance. (Left) Away from the main diagonal in the $(\epsilon_{ac},\epsilon^j_{dc})$ plane the differential resistance has minima and maxima described by conditions (4.36). (Right) Close to the main diagonal $\epsilon_{ac} \simeq \epsilon^j_{dc}$, the differential resistance becomes a function of the sum $\epsilon_{ac}+\epsilon^j_{dc}$.

We now analyze the differential resistance at large values of direct current, $\epsilon^j_{dc} \gg 1$. In this limit the displacement contribution dominates over the inelastic contribution. Therefore, we keep only $F_d(\epsilon^j_{dc},\epsilon_{ac})$ and find

$$\frac{\delta\rho}{\rho_D} = \frac{(4\lambda)^2\tau_{tr}}{\pi\tau_\pi}\Bigg[(1-2\mathcal{P}_\omega)\cos 2\pi\epsilon_{dc} + 2\mathcal{P}_\omega\Bigg(\cos 2\pi\epsilon_{dc}\cos 2\pi\epsilon_{ac} - \frac{\epsilon_{ac}}{\epsilon_{dc}}\sin 2\pi\epsilon_{dc}\sin 2\pi\epsilon_{ac}\Bigg)\Bigg]. \tag{4.35}$$

The first term in square brackets coincides with the result of Ref. 26 at $\mathcal{P}_\omega=0$. At finite $\mathcal{P}_\omega$ the factor $(1-2\mathcal{P}_\omega)$ represents the impurity potential renormalization due to the virtual absorption and emission of photons. These radiative corrections tend to suppress the dark resistivity and can be interpreted as motional narrowing or averaging out the electrostatic potential of impurities in the presence of an oscillating electric field. The other two terms in Eq. (4.35) describe the effect of the combined scattering off disorder in mixed constant and oscillating electric fields when the real microwave photons are absorbed or emitted.

We briefly discuss the properties of Eq. (4.35) as a function of two parameters ϵ^j_{dc} and ϵ_{ac}. It is plotted in Fig. 4 as a grayscale contour plot. This function exhibits a series of maxima and minima in $(\epsilon_{ac},\epsilon^j_{dc})$ plane. In general, for $\epsilon_{ac} > \epsilon^j_{dc}$ the local maxima and minima of the function defined by Eq. (4.35) are located at

$$(\epsilon_{ac},\epsilon^j_{dc})^{\max} = (m \pm 1/4, n \mp 1/4),$$

$$(\epsilon_{ac},\epsilon^j_{dc})^{\min} = (m \pm 1/4, n \pm 1/4). \tag{4.36}$$

We further notice that in the region of parameters $|\epsilon_{ac}-\epsilon^j_{dc}| \lesssim |\epsilon_{ac}+\epsilon^j_{dc}|$, namely, not too far from the main diagonal in the two-dimensional plain $(\epsilon_{ac},\epsilon_{dc})$, result (4.35) reduces to

$$\frac{\delta\rho}{\rho_D} \approx \frac{(4\lambda)^2\tau_{\rm tr}}{\pi\tau_\pi}[(1-2\mathcal{P}_\omega)\cos 2\pi\epsilon_{\rm dc}^j + 2\mathcal{P}_\omega \cos 2\pi(\epsilon_{\rm dc}^j + \epsilon_{\rm ac})]. \tag{4.37}$$

Although a direct addition of two different parameters $\epsilon_{\rm dc}^j$ and $\epsilon_{\rm ac}$ has no physical meaning, we believe that, in the considered region $\epsilon_{\rm dc} \simeq \epsilon_{\rm ac}$, it is the second term which is responsible for apparent structure[27] of the differential resistance as a function of the sum $\epsilon_{\rm ac}+\epsilon_{\rm dc}^j$ [see Fig. 4(b)].

V. NONLINEAR MAGNETORESISTANCE BEYOND THE BILINEAR RESPONSE IN MICROWAVE FIELD

In this section we extend the analysis of the magnetotransport at $\epsilon_{\rm dc} \gg 1$ to the case of arbitrary strength of microwave radiation. We again consider the case of relatively high microwave frequency $\omega \gg 1/\tau_{\rm tr}$. The last condition ensures that the oscillating in the variable $(t+t')/2$ part of the distribution function is small compared to the stationary one. We, therefore, can assume that the distribution function is homogeneous in time, $f_{t,t'}=f_{t-t'}$. Beyond the bilinear response, Eq. (3.50), obtained as the expansion of Eq. (3.25) in powers of $\mathcal{P}_\omega$, is no longer valid. We perform the time averaging of the exact collision kernel [Eq. (3.25)] over the microwave oscillation period,

$$\overline{\hat{\mathcal{K}}}_{t,t';\varphi}\{F(\varphi)\} = \int \frac{d\varphi'}{2\pi}F(\varphi')\frac{e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta_{t-t'}^{\rm dc}}}{\tau_{\varphi-\varphi'}}\overline{e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta_{t,t'}^{\rm ac}}}. \tag{5.1}$$

For the circular polarization the displacement due to the microwave field $\zeta_{t,t'}^{\rm ac}$ is given by Eq. (3.47). The time averaging in Eq. (5.1) results in

$$\overline{e^{ip_F(\boldsymbol{n}_\varphi-\boldsymbol{n}_{\varphi'})\zeta_{t,t'}^{\rm ac}}} = J_0\left(Q_{\varphi-\varphi'}\sin\frac{\omega(t-t')}{2}\right) \tag{5.2}$$

with $J_0(x)$ being the Bessel function and

$$Q_{\varphi-\varphi'} = 4\sqrt{\mathcal{P}_\omega}\left|\sin\frac{\varphi-\varphi'}{2}\right|. \tag{5.3}$$

The time-averaged collision kernel takes the form

$$\overline{\hat{\mathcal{K}}}_{t,t';\varphi}\{F(\varphi)\} = \int \frac{d\varphi'}{2\pi}F(\varphi')\frac{e^{iW_{\varphi\varphi'}(t-t')}}{\tau_{\varphi-\varphi'}}J_0\left(Q_{\varphi-\varphi'}\sin\frac{\omega(t-t')}{2}\right). \tag{5.4}$$

The substitution of Eq. (5.4) to Eq. (3.31) leads to the following expressions for the out collision term:

$$\overline{\rm St}_{\rm out}\{f\}_{t,t'} = -\int \frac{d\varphi'}{2\pi}\frac{f_{t,t'}}{\tau_{\varphi-\varphi'}} - \int \frac{d\varphi'}{2\pi}\frac{\lambda}{\tau_{\varphi-\varphi'}}J_0(Q_{\varphi-\varphi'}\sin\pi\epsilon_{\rm ac}) \times[e^{iW_{\varphi\varphi'}T_c}f_{t-T_c,t'} + e^{-iW_{\varphi\varphi'}T_c}f_{t,t'-T_c}]. \tag{5.5}$$

The scattering-in term Eq. (3.36) reads

$$\overline{\rm St}_{\rm in}\{f\}_{t,t'} = \int \frac{d\varphi'}{2\pi}\frac{e^{iW_{\varphi\varphi'}(t-t')}}{\tau_{\varphi-\varphi'}}J_0\left(Q_{\varphi-\varphi'}\sin\frac{\omega(t-t')}{2}\right) \times(f_{t,t'} - \lambda f_{t-T_c,t'} - \lambda f_{t,t'-T_c}). \tag{5.6}$$

We rewrite the kinetic equation Eq. (3.42) for homogeneous in time distribution function in the energy representation

$$\omega_c\frac{\partial}{\partial\varphi}f_{\varepsilon;\varphi} = {\rm St}_{\rm dis}\{f_{\varepsilon;\varphi}\}_{\varepsilon;\varphi}. \tag{5.7}$$

The full collision integral for electron scattering off disorder, ${\rm St}_{\rm dis}\{f_{\varepsilon;\varphi}\}_{\varepsilon;\varphi}$, is a sum of the scattering-in and scattering-out terms given by Eqs. (5.5) and (5.6) [see Eq. (3.28)].

We discuss the nonlinear transport regime $\epsilon_{\rm dc} \gg 1$. In this case it is sufficient to consider the displacement contribution to the nonlinear current. To calculate the displacement contribution to the electric current, we substitute the equilibrium distribution function Eq. (2.10) to the collision integral in Eq. (5.7). Combining Eqs. (5.5) and (5.6) and writing the result in energy representation, we obtain

$${\rm St}_{\rm dis}\{f_T(\varepsilon)\}_{\varepsilon;\varphi} = \int \frac{d\varphi'}{2\pi}\frac{K_0 - 2\lambda\,{\rm Re}\{e^{i(\varepsilon+W_{\varphi\varphi'})T_c}K_1\}}{\tau_{\varphi-\varphi'}}. \tag{5.8}$$

The first kernel,

$$K_0 = J_0\left(Q_{\varphi-\varphi'}\sin\frac{i\omega\partial_\varepsilon}{2}\right)f_T(\varepsilon+W_{\varphi\varphi'}) - f_T(\varepsilon), \tag{5.9}$$

is the part of the collision integral to the zero order in λ. In particular, it describes the classical Drude conductivity. Although the microwave radiation complicates the form of K_0, we notice that this kernel can be represented in terms of the series expansion in powers of $\partial/\partial\varepsilon$, applied to the Fermi distribution function. The observables, such as electric current, are determined by energy integrals. Because the integrals of all derivatives of $f_T(\varepsilon)$ of the second order and higher vanish, we do not expect any effect of microwave radiation within our model on the conductivity to the zeroth order in λ.

The second term in Eq. (5.7) is

$$K_1 = J_0\left(Q_{\varphi-\varphi'}\sin\left[\frac{i\omega\partial_\epsilon}{2} - \pi\epsilon_{\rm ac}\right]\right)f_T(\varepsilon+W_{\varphi\varphi'}) - J_0(Q_{\varphi-\varphi'}\sin\pi\epsilon_{\rm ac})f_T(\epsilon). \tag{5.10}$$

In the high-temperature limit, $T \gg |e|ER_c$ and $T \gg \omega$, Eq. (5.10) can be reduced to

$$K_1 = \frac{\partial f_T(\varepsilon)}{\partial\epsilon}\left[J_0(Q_{\varphi-\varphi'}\sin\pi\epsilon_{\rm ac})W_{\varphi\varphi'} + \frac{i\omega Q_{\varphi-\varphi'}}{2}J_1(Q_{\varphi-\varphi'}\sin\pi\epsilon_{\rm ac})\cos\pi\epsilon_{\rm ac}\right]. \tag{5.11}$$

Similarly to the case of the small power, we look for the correction to the distribution function in the form

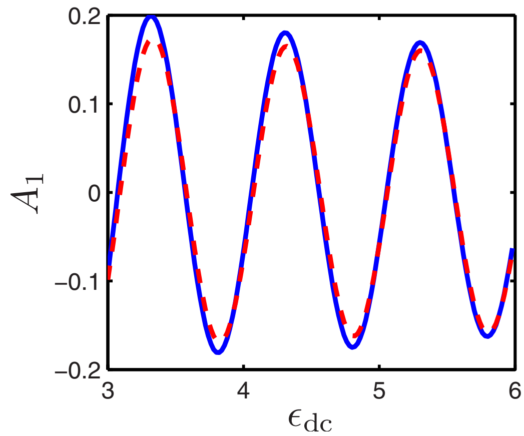

FIG. 5. (Color online) The function A_1 evaluated by (a) numerical integration in Eq. (5.13), solid line (blue), and (b) by the stationary phase approximation applied to Eq. (5.13), dashed line (red) for $\mathcal{P}_\omega=8.25$ and $\epsilon_{ac}=2.25$.

$$\delta f(\varepsilon,\varphi)=\delta f_{cl}(\varepsilon,\varphi)+\lambda\partial_\varepsilon f_T A_1 \cos\frac{2\pi\varepsilon}{\omega_c}\cos\varphi. \quad (5.12)$$

Substituting $\delta f(\varepsilon,\varphi)$ to Eq. (5.7), we find that $\delta f_{cl}(\varepsilon,\varphi)$ is given by a sum of Eq. (4.3) and higher-order derivatives of $f_T(\varepsilon)$, as discussed below Eq. (5.9). The term of the first order in λ determines the value of coefficient A_1:

$$A_1=\frac{1}{\omega_c}\int\frac{d\varphi d\varphi'}{\pi^2}\frac{\sin\varphi}{\tau_{\varphi-\varphi'}}\left[\cos\frac{2\pi W_{\varphi\varphi'}}{\omega_c}J_0(Q_{\varphi-\varphi'}\sin\pi\epsilon_{ac})W_{\varphi\varphi'} - \sin\frac{2\pi W_{\varphi\varphi'}}{\omega_c}\frac{\omega Q_{\varphi-\varphi'}}{2}J_1(Q_{\varphi-\varphi'}\sin\pi\epsilon_{ac})\cos\pi\epsilon_{ac}\right]. \quad (5.13)$$

Our analysis in this section is applicable for $\epsilon_{dc}\gtrsim 1$, when we can utilize the saddle-point approximation, to perform angular integrations in Eq. (5.13). Angles φ and φ' which make the phase of the integrand stationary are $\varphi=\pm\pi/2$ and $\varphi'=\mp\pi/2$ and correspond to $2R_c$ jumps along the electric-field direction (see Sec. II). For $\sqrt{\mathcal{P}_\omega}\lesssim\epsilon_{dc}$ the angular integrations in Eq. (5.13) result in

$$A_1=-\frac{4}{\pi^2\tau_\pi}\sin 2\pi\epsilon_{dc}J_0(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac}) - \frac{8}{\pi^2\tau_\pi}\frac{\epsilon_{ac}}{\epsilon_{dc}}\cos 2\pi\epsilon_{dc}\cos\pi\epsilon_{ac}\sqrt{\mathcal{P}_\omega}J_1(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac}). \quad (5.14)$$

The result of numerical evaluation of the coefficient A_1, Eq. (5.13), and its approximation, Eq. (5.14), are shown in Fig. 5. The stationary phase approximation progressively improves as the parameter ϵ_{dc} increases.

The nonlinear contribution to the electric current is given by Eq. (4.10) in terms of coefficients A_1, Eq. (5.13), and $A_2=0$. Representing the nonlinear contribution to the current in the form of Eq. (4.11), we write

$$F(\epsilon_{dc},\epsilon_{ac})=-\frac{\tau_{tr}}{\epsilon_{dc}}A_1. \quad (5.15)$$

The function $F(\epsilon_{dc},\epsilon_{ac})$ found from Eqs. (5.14) and (5.15) reads

$$F(\epsilon_{dc},\epsilon_{ac})=\frac{4\tau_{tr}}{\pi^2\tau_\pi\epsilon_{dc}}\left[\sin 2\pi\epsilon_{dc}J_0(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac}) + \frac{2\epsilon_{ac}}{\epsilon_{dc}}\cos 2\pi\epsilon_{dc}\cos\pi\epsilon_{ac}\sqrt{\mathcal{P}_\omega}J_1(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac})\right]. \quad (5.16)$$

The oscillatory correction to the differential magnetoresistance is obtained by substituting Eq. (5.16) to Eq. (4.33),

$$\frac{\delta\rho(j)}{\rho_D}=\frac{(4\lambda)^2\tau_{tr}}{\pi\tau_\pi}\left[\cos 2\pi\epsilon_{dc}J_0(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac}) - \frac{2\epsilon_{ac}}{\epsilon_{dc}}\sin 2\pi\epsilon_{dc}\cos\pi\epsilon_{ac}\sqrt{\mathcal{P}_\omega}J_1(4\sqrt{\mathcal{P}_\omega}\sin\pi\epsilon_{ac})\right]. \quad (5.17)$$

Relation (5.17) reduces to Eq. (4.35) obtained earlier in the weak power limit as expected.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have presented a comprehensive theory of an out of equilibrium two-dimensional electron system (2DES) in a magnetic field in the case when the disordered potential has finite scattering amplitude on an arbitrary angle. Then, we applied this theory to analyze electron transport in the presence of constant and oscillating in-plane electric fields. We showed that the electric current has an oscillating component as a function of the strength of the constant electric field and of the frequency of an oscillating electric field. We have investigated the position of maxima and minima of the differential resistance on the plane of parameters ϵ_{dc} and ϵ_{ac} and found qualitative agreement with experiments of Refs. 27 and 28.

Actual value of the differential resistance as a function of the current through a 2DES sample depends on a number of different scattering rates, including the full quantum-scattering rate, the transport scattering rate, and the back-scattering rate. Our analytical results may be applied to the experimentally measured differential resistance to evaluate these scattering rates and to obtain a detailed picture of the origin and structure of the disorder in high-mobility 2DES samples. In particular, we would like to emphasize that the behavior of the differential resistance at large currents in experiments of Refs. 21, 27, and 28 suggests that the disordered potential has a noticeably strong short-range component, responsible for the finite backscattering rate.

The presence of the short-range disorder also modifies the current behavior in weak constant electric fields and at weak power of an oscillating electric field. In particular, there are two competing contributions to the current. One contribution originates from the modification of the isotropic component of the electron distribution function by electric fields,[18] and we refer to this contribution as an "inelastic" contribution. The other contribution is due to the modification of electron-scattering rate off disorder by electric fields, known as the displacement contribution.[15–17,30] While in smooth disorder in weak fields the inelastic contribution dominates in the whole range of temperatures at which the nonlinear current is expected to survive, the short-range disorder may make these two contributions comparable. As a result, the strong temperature dependence of the nonlinear current in weak fields is expected only in samples with sufficiently weak short-range disorder.

We also studied the dependence of nonlinear current on the applied power of oscillating electric field. Expression (5.17) obtained in the nonlinear transport regime $\epsilon_{\rm dc}\gg 1$ is not limited to the small microwave radiation power. This expression shows that in strong microwave fields the nonlinear current has a different dependence on parameters $\epsilon_{\rm ac}$ and $\epsilon_{\rm dc}$, Eq. (1.1), which qualitatively differs from the corresponding expressions in the weak-power limit. We believe that the study of nonlinear current at strong-radiation power may bring additional opportunities for study of microscopic characteristics of high-mobility electron systems.

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APPENDIX: CALCULATIONS OF THE ELECTRON DISTRIBUTION FUNCTION

In this appendix we give the details of the calculation of the coefficients I, A_1, and A_2 of the distribution function in the form of Eq. (4.4). Substituting Eq. (4.4) to the kinetic Eq. (3.42) with the collision terms specified by Eqs. (3.53) and (3.54), we obtain the following system of equations:

$$
\begin{aligned}
I\left[\frac{1}{\tau_{\rm ee}(\varepsilon)}+\frac{1}{\tau_0}\right]\sin\frac{2\pi\varepsilon}{\omega_c}
&=\langle K_{\varphi\varphi'}(\varepsilon)-2\mathcal{P}_\omega[\sin^2\pi\epsilon_{\rm ac}\bar K_{\varphi\varphi'}(\varepsilon)\\
&\quad+\omega\sin 2\pi\epsilon_{\rm ac}\bar M_{\varphi\varphi'}(\varepsilon)]\ +I[M_{\varphi\varphi'}(\varepsilon)\\
&\quad-2\mathcal{P}_\omega\sin^2\pi\epsilon_{\rm ac}\bar M_{\varphi\varphi'}(\varepsilon)]\rangle,
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
-\frac{\omega_c A_1}{2}\cos\frac{2\pi\varepsilon}{\omega_c}&=\langle\sin\varphi\{K_{\varphi\varphi'}(\varepsilon)-2\mathcal{P}_\omega[\sin^2\pi\epsilon_{\rm ac}\bar K_{\varphi\varphi'}(\varepsilon)\\
&\quad-\omega\sin 2\pi\epsilon_{\rm ac}\bar M_{\varphi\varphi'}(\varepsilon)]+I[M_{\varphi\varphi'}(\varepsilon)\\
&\quad-2\mathcal{P}_\omega\sin^2\pi\epsilon_{\rm ac}\bar M_{\varphi\varphi'}(\varepsilon)]\}\rangle,
\end{aligned}
\tag{A2}
$$

and

$$
\begin{aligned}
\frac{\omega_c A_2}{4}&=I\Big\langle\sin\varphi\cos\frac{2\pi(\varepsilon+W_{\varphi\varphi'})}{\omega_c}\left(M_{\varphi\varphi'}(\varepsilon)-\frac{\sin\varepsilon}{\tau_{\varphi-\varphi'}}\right)\\
&\quad-\frac{\mathcal{P}_\omega}{2}\sum_\pm\left[\cos\frac{2\pi(\varepsilon+W_{\varphi\varphi'})}{\omega_c}\left(\bar M_{\varphi\varphi'}(\varepsilon)-\frac{\sin\varepsilon}{\bar\tau_{\varphi-\varphi'}}\right)\right.\\
&\quad\left.-\cos\frac{2\pi(\varepsilon+W_{\varphi\varphi'}\pm\omega)}{\omega_c}\left(\bar M_{\varphi\varphi'}(\varepsilon\pm\omega)-\frac{\sin\varepsilon}{\bar\tau_{\varphi-\varphi'}}\right)\right]\Big\rangle,
\end{aligned}
\tag{A3}
$$

where $\langle\cdots\rangle$ stands for the averaging over angular variables φ and φ'. Here we have introduced integral kernels

$$K_{\varphi\varphi'}(\varepsilon)=-2\cos\frac{2\pi(\varepsilon+W_{\varphi\varphi'})}{\omega_c}\frac{W_{\varphi\varphi'}}{\tau_{\varphi-\varphi'}},\tag{A4a}$$

$$M_{\varphi\varphi'}(\varepsilon)=\frac{\sin[2\pi(\varepsilon+W_{\varphi\varphi'})/\omega_c]}{\tau_{\varphi-\varphi'}}.\tag{A4b}$$

The kernels $\bar K_{\varphi\varphi'}(\varepsilon)$ and $\bar M_{\varphi\varphi'}(\varepsilon)$ are obtained from Eqs. (A4a) and (A4b), respectively, by replacing $\tau\to\bar\tau$ given by Eq. (2.5). Keeping only the energy independent part in Eq. (A3) surviving the subsequent energy integration, we rewrite it as

$$\frac{\omega_c A_2}{2}=I\langle\sin\varphi[M_{\varphi\varphi'}(0)-2\mathcal{P}_\omega\sin^2\pi\epsilon_{\rm ac}\bar M_{\varphi\varphi'}(0)]\rangle.\tag{A5}$$

For the angular averages appearing in Eqs. (A1), (A2), and (A5), we have the following expressions:

$$\langle K_{\varphi\varphi'}(\epsilon)\rangle=\frac{2eER_c}{\pi}\gamma'(\epsilon_{\rm dc})\sin\frac{2\pi\epsilon}{\omega_c},\tag{A6a}$$

$$\langle M_{\varphi\varphi'}(\epsilon)\rangle=\gamma(\epsilon_{\rm dc})\sin\frac{2\pi\epsilon}{\omega_c},\tag{A6b}$$

$$\langle\sin\varphi K_{\varphi\varphi'}(\epsilon)\rangle=\frac{eER_c\gamma''(\epsilon_{\rm dc})}{\pi^2}\cos\frac{2\pi\epsilon}{\omega_c},\tag{A6c}$$

$$\langle\sin\varphi M_{\varphi\varphi'}(\epsilon)\rangle=\frac{\gamma'(\epsilon_{\rm dc})}{2\pi}\cos\frac{2\pi\epsilon}{\omega_c}.\tag{A6d}$$

In Eq. (A6) the function $\gamma(\epsilon_{\rm dc})$ has been introduced in Eq. (4.6a). The angular averages involving bared functions $\bar K$ and $\bar M$ are given by Eq. (A6) with $\bar\gamma(\epsilon_{\rm dc})$, Eq. (4.6b), replacing $\gamma(\epsilon_{\rm dc})$. Relation (A6) allows us to perform the angular integrations in Eqs. (A1)–(A3) leading to expressions (4.5), (4.7), and (4.8) of the main text.

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